1.3 Transformations of Function Graphs

Essential Question: What are the ways you can transform the graph of the function $f(x)$?

Explore 1  Investigating Translations of Function Graphs

You can transform the graph of a function in various ways. You can translate the graph horizontally or vertically, you can stretch or compress the graph horizontally or vertically, and you can reflect the graph across the $x$-axis or the $y$-axis. How the graph of a given function is transformed is determined by the way certain numbers, called parameters, are introduced in the function.

The graph of $f(x)$ is shown. Use this graph for the exploration.

First graph $g(x) = f(x) + k$ where $k$ is the parameter. Let $k = 4$ so that $g(x) = f(x) + 4$. Complete the input-output table and then graph $g(x)$. In general, how is the graph of $g(x) = f(x) + k$ related to the graph of $f(x)$ when $k$ is a positive number?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f(x) + 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-2$</td>
<td>$2$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
<td>$6$</td>
</tr>
<tr>
<td>$3$</td>
<td>$-2$</td>
<td>$2$</td>
</tr>
<tr>
<td>$5$</td>
<td>$2$</td>
<td>$6$</td>
</tr>
</tbody>
</table>

Now try a negative value of $k$ in $g(x) = f(x) + k$. Let $k = -3$ so that $g(x) = f(x) - 3$. Complete the input-output table and then graph $g(x)$ on the same grid. In general, how is the graph of $g(x) = f(x) + k$ related to the graph of $f(x)$ when $k$ is a negative number?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f(x) - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-2$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$1$</td>
<td>$2$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$3$</td>
<td>$-2$</td>
<td>$-5$</td>
</tr>
<tr>
<td>$5$</td>
<td>$2$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>
Now graph \( g(x) = f(x - h) \) where \( h \) is the parameter. Let \( h = 2 \) so that \( g(x) = f(x - 2) \). Complete the mapping diagram and then graph \( g(x) \). (To complete the mapping diagram, you need to find the inputs for \( g \) that produce the inputs for \( f \) after you subtract 2. Work backward from the inputs for \( f \) to the missing inputs for \( g \) by adding 2.) In general, how is the graph of \( g(x) = f(x - h) \) related to the graph of \( f(x) \) when \( h \) is a positive number?

**Horizontal Shift**

**Make a Conjecture** How would you expect the graph of \( g(x) = f(x - h) \) to be related to the graph of \( f(x) \) when \( h \) is a negative number?

**Horizontal Shift** \( h \) units left

**Reflect**

1. Suppose a function \( f(x) \) has a domain of \([x_1, x_3]\) and a range of \([y_1, y_2]\). When the graph of \( f(x) \) is translated vertically \( k \) units where \( k \) is either positive or negative, how do the domain and range change?

2. Suppose a function \( f(x) \) has a domain of \([x_1, x_3]\) and a range of \([y_1, y_2]\). When the graph of \( f(x) \) is translated horizontally \( h \) units where \( h \) is either positive or negative, how do the domain and range change?

3. You can transform the graph of \( f(x) \) to obtain the graph of \( g(x) = f(x - h) + k \) by combining transformations. Predict what will happen by completing the table.

<table>
<thead>
<tr>
<th>Sign of ( h )</th>
<th>Sign of ( k )</th>
<th>Transformations of the Graph of ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>Translate right ( h ) units and up ( k ) units.</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
Explore 2  Investigating Stretches and Compressions of Function Graphs

In this activity, you will consider what happens when you multiply by a positive parameter inside or outside a function. Throughout, you will use the same function \( f(x) \) that you used in the previous activity.

A. First graph \( g(x) = a \cdot f(x) \) where \( a \) is the parameter. Let \( a = 2 \) so that \( g(x) = 2f(x) \). Complete the input-output table and then graph \( g(x) \). In general, how is the graph of \( g(x) = a \cdot f(x) \) related to the graph of \( f(x) \) when \( a \) is greater than 1?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( 2f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Vertical Stretch \( A \) units**

B. Now try a value of \( a \) between 0 and 1 in \( g(x) = a \cdot f(x) \). Let \( a = \frac{1}{2} \) so that \( g(x) = \frac{1}{2}f(x) \). Complete the input-output table and then graph \( g(x) \). In general, how is the graph of \( g(x) = a \cdot f(x) \) related to the graph of \( f(x) \) when \( a \) is a number between 0 and 1?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( \frac{1}{2}f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Vertical Compression \( A \) units**
Now graph \( g(x) = f \left( \frac{1}{b} \cdot x \right) \) where \( b \) is the parameter. Let \( b = 2 \) so that \( g(x) = f \left( \frac{1}{2} x \right) \). Complete the mapping diagram and then graph \( g(x) \). (To complete the mapping diagram, you need to find the inputs for \( g \) that produce the inputs for \( f \) after you multiply by \( \frac{1}{2} \). Work backward from the inputs for \( f \) to the missing inputs for \( g \) by multiplying by 2.) In general, how is the graph of \( g(x) = f(\frac{1}{b} x) \) related to the graph of \( f(x) \) when \( b \) is a number greater than 1?

\[
\begin{array}{|c|c|}
\hline
\text{Input for } g & \text{Input for } f \\
\hline
\frac{1}{2} & 1 \\
2 & 3 \\
10 & 5 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Output for } f & \text{Output for } g \\
\hline
2 & 2 \\
2 & 2 \\
2 & 2 \\
\hline
\end{array}
\]

\[
g(10) = f \left( \frac{1}{2} \cdot 10 \right) = f(5) = 2
\]

**Horizontal Stretch of 2 units**

**Make a Conjecture** How would you expect the graph of \( g(x) = f \left( \frac{1}{b} \cdot x \right) \) to be related to the graph of \( f(x) \) when \( b \) is a number between 0 and 1? \( g(x) = \sigma(\text{BX}) \)

**Horizontal Compression of \( B \) units**

**Reflect**

4. Suppose a function \( f(x) \) has a domain of \([x_1, x_2]\) and a range of \([y_1, y_2]\). When the graph of \( f(x) \) is stretched or compressed vertically by a factor of \( a \), how do the domain and range change?

5. You can transform the graph of \( f(x) \) to obtain the graph of \( g(x) = a \cdot f(x-h) + k \) by combining transformations. Predict what will happen by completing the table.

<table>
<thead>
<tr>
<th>Value of ( a )</th>
<th>Transformations of the Graph of ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a &gt; 1 )</td>
<td>Stretch vertically by a factor of ( a ), and translate ( h ) units horizontally and ( k ) units vertically.</td>
</tr>
<tr>
<td>( 0 &lt; a &lt; 1 )</td>
<td></td>
</tr>
</tbody>
</table>

6. You can transform the graph of \( f(x) \) to obtain the graph of \( g(x) = f \left( \frac{1}{b}(x - h) \right) + k \) by combining transformations. Predict what will happen by completing the table.

<table>
<thead>
<tr>
<th>Value of ( b )</th>
<th>Transformations of the Graph of ( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b &gt; 1 )</td>
<td>Stretch horizontally by a factor of ( b ), and translate ( h ) units horizontally and ( k ) units vertically.</td>
</tr>
<tr>
<td>( 0 &lt; b &lt; 1 )</td>
<td></td>
</tr>
</tbody>
</table>
Investigating Reflections of Function Graphs

When the parameter in a stretch or compression is negative, another transformation called a reflection is introduced. Examining reflections will also tell you whether a function is an even function or an odd function. An even function is one for which \( f(-x) = f(x) \) for all \( x \) in the domain of the function, while an odd function is one for which \( f(-x) = -f(x) \) for all \( x \) in the domain of the function. A function is not necessarily even or odd; it can be neither.

A. First graph \( g(x) = a \cdot f(x) \) where \( a = -1 \). Complete the input-output table and then graph \( g(x) = -f(x) \). In general, how is the graph of \( g(x) = -f(x) \) related to the graph of \( f(x) \)?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( -f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Reflection Across X Axis**

B. Now graph \( g(x) = f\left(\frac{1}{b} \cdot x\right) \) where \( b = -1 \). Complete the input-output table and then graph \( g(x) = f(-x) \). In general, how is the graph of \( g(x) = f(-x) \) related to the graph of \( f(x) \)?

<table>
<thead>
<tr>
<th>Input for ( g )</th>
<th>Input for ( f )</th>
<th>Output for ( f )</th>
<th>Output for ( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Reflection Across Y Axis**

Reflect

7. **Discussion** Suppose a function \( f(x) \) has a domain of \([x_1, x_2]\) and a range of \([y_1, y_2]\). When the graph of \( f(x) \) is reflected across the x-axis, how do the domain and range change?

8. For a function \( f(x) \), suppose the graph of \( f(-x) \), the reflection of the graph of \( f(x) \) across the y-axis, is identical to the graph of \( f(x) \). What does this tell you about \( f(x) \)? Explain.

9. Is the function whose graph you reflected across the axes in Steps A and B an even function, an odd function, or neither? Explain.
### Explain 1

**Transforming the Graph of the Parent Quadratic Function**

You can use transformations of the graph of a basic function, called a parent function, to obtain the graph of a related function. To do so, focus on how the transformations affect reference points on the graph of the parent function.

For instance, the parent quadratic function is \( f(x) = x^2 \). The graph of this function is a U-shaped curve called a parabola with a turning point, called a vertex, at \((0, 0)\). The vertex is a useful reference point, as are the points \((-1, 1)\) and \((1, 1)\).

### Example 1

Describe how to transform the graph of \( f(x) = x^2 \) to obtain the graph of the related function \( g(x) \). Then draw the graph of \( g(x) \).

\[
g(x) = \frac{1}{3}f(x - 2)^2 - 4\]

<table>
<thead>
<tr>
<th>Parameter and Its Value</th>
<th>Effect on the Parent Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a = -3)</td>
<td>vertical stretch of the graph of ( f(x) ) by a factor of 3 and a reflection across the x-axis</td>
</tr>
<tr>
<td>(b = 1)</td>
<td>Since ( b = 1 ), there is no horizontal stretch or compression.</td>
</tr>
<tr>
<td>(h = 2)</td>
<td>horizontal translation of the graph of ( f(x) ) to the right 2 units</td>
</tr>
<tr>
<td>(k = -4)</td>
<td>vertical translation of the graph of ( f(x) ) down 4 units</td>
</tr>
</tbody>
</table>

Applying these transformations to a point \((x, y)\) on the parent graph results in the point \((x + 2, -3y - 4)\). The table shows what happens to the three reference points on the graph of \( f(x) \).

<table>
<thead>
<tr>
<th>Point on the Graph of ( f(x) )</th>
<th>Corresponding Point on ( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, 1))</td>
<td>((-1 + 2, -3(1) - 4) = (1, -7))</td>
</tr>
<tr>
<td>((0, 0))</td>
<td>((0 + 2, -3(0) - 4) = (2, -4))</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>((1 + 2, -3(1) - 4) = (3, -7))</td>
</tr>
</tbody>
</table>

Use the transformed reference points to graph \( g(x) \).
1.3 Transformations of Function Graphs

\[ g(x) = f \left( \frac{1}{2}(x - h) \right) + k \]

\[ g(x) = f \left( \frac{1}{2}(x + 5) \right) + 2 \]

<table>
<thead>
<tr>
<th>Parameter and Its Value</th>
<th>Effect on the Parent Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 1 )</td>
<td>Since ( a = 1 ), there is no vertical stretch, no vertical compression, and no reflection across the x-axis.</td>
</tr>
<tr>
<td>( b = 2 )</td>
<td>The parent graph is stretched/compressed horizontally by a factor of ( \frac{1}{2} ). There is no reflection across the y-axis.</td>
</tr>
<tr>
<td>( h = -5 )</td>
<td>The parent graph is translated ( -5 ) units horizontally/vertically.</td>
</tr>
<tr>
<td>( k = 2 )</td>
<td>The parent graph is translated ( 2 ) units horizontally/vertically.</td>
</tr>
</tbody>
</table>

Applying these transformations to a point on the parent graph results in the point \((2x - 5, y + 2)\). The table shows what happens to the three reference points on the graph of \( f(x) \).

Applying these transformations to a point on the parent graph results in the point \((2x - 5, y + 2)\). The table shows what happens to the three reference points on the graph of \( f(x) \).

<table>
<thead>
<tr>
<th>Point on the Graph of ( f(x) )</th>
<th>Corresponding Point on the Graph of ( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, 1))</td>
<td>((2(-1) - 5, 1 + 2) = (-7, 3))</td>
</tr>
<tr>
<td>((0, 0))</td>
<td>((2(0) - 5, 0 + 2) = (-5, 2))</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>((2(1) - 5, 1 + 2) = (-3, 3))</td>
</tr>
</tbody>
</table>

Use the transformed reference points to graph \( g(x) \).

---

10. Is the function \( f(x) = x^2 \) an even function, an odd function, or neither? Explain.

11. The graph of the parent quadratic function \( f(x) = x^2 \) has the vertical line \( x = 0 \) as its axis of symmetry. Identify the axis of symmetry for each of the graphs of \( g(x) \) in Parts A and B. Which transformation(s) affect the location of the axis of symmetry?

   - **A**: \( x = 2 \)
   - **B**: \( x = -5 \)
Your Turn

12. Describe how to transform the graph of \( f(x) = x^2 \) to obtain the graph of the related function \( g(x) = f(-4(x - 3)) + 1 \). Then draw the graph of \( g(x) \).

\[(1, 1) \Rightarrow (2.75, 2)\]
\[(0, 0) \Rightarrow (3, 1)\]
\[(-1, 1) \Rightarrow (3.75, 2)\]

**Explain 2**  
**Modeling with a Quadratic Function**

You can model real-world objects that have a parabolic shape using a quadratic function. In order to fit the function’s graph to the shape of the object, you will need to determine the values of the parameters in the function \( g(x) = a \cdot f \left( \frac{1}{b} (x - h) \right) + k \) where \( f(x) = x^2 \). Note that because \( f(x) \) is simply a squaring function, it’s possible to pull the parameter \( b \) outside the function and combine it with the parameter \( a \). Doing so allows you to model real-objects using \( g(x) = a \cdot f(x - h) + k \), which has only three parameters.

When modeling real-world objects, remember to restrict the domain of \( g(x) = a \cdot f(x - h) + k \) to values of \( x \) that are based on the object’s dimensions.

**Example 2**

An old stone bridge over a river uses a parabolic arch for support. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the arch is point \( C \). Find a quadratic function that models the arch, and state the function’s domain.
1.3 Transformations of Function Graphs

Analyze Information
Identify the important information.
- The shape of the arch is a **parabola**.
- The vertex of the parabola is **(27, -5)**.
- Two other points on the parabola are **(2, -20)** and **(52, -20)**.

Formulate a Plan
You want to find the values of the parameters \(a, h,\) and \(k\) in \(g(x) = a \cdot f(x - h) + k\) where \(f(x) = x^2\). You can use the coordinates of point \(C\) to find the values of \(h\) and \(k\). Then you can use the coordinates of one of the other points to find the value of \(a\).

Solve
The vertex of the graph of \(g(x)\) is point \(C\), and the vertex of the graph of \(f(x)\) is the origin. Point \(C\) is the result of translating the origin 27 units to the right and 5 units down. This means that \(h = 27\) and \(k = -5\). Substituting these values into \(g(x)\) gives \(g(x) = a \cdot f(x - 27) - 5\). Now substitute the coordinates of point \(B\) into \(g(x)\) and solve for \(a\).

\[
\begin{align*}
g(x) &= a \cdot f(x - 27) - 5 \\
g(52) &= a \cdot f(52 - 27) - 5 \\
-20 &= a \cdot f(25) - 5 \\
-20 &= a \cdot (25) - 5 \\
-20 &= a(625) - 5 \\
-19 &= a(625) \\
a &= \frac{-19}{625} \\
\end{align*}
\]

Substitute the value of \(a\) into \(g(x)\).

\[
g(x) = \frac{-19}{625} f(x - 27) - 5
\]

The arch exists only between points \(A\) and \(B\), so the domain of \(g(x)\) is \(\{x | 2 \leq x \leq 52\}\).

Justify and Evaluate
To justify the answer, verify that \(g(2) = -20\).

\[
\begin{align*}
g(x) &= \frac{-19}{625} f(x - 27) - 5 \\
g(2) &= \frac{-19}{625} f(2 - 27) - 5 \\
&= \frac{-19}{625} f(-25) - 5 \\
&= \frac{-19}{625} \cdot (-25) - 5 \\
&= -20 \quad \checkmark
\end{align*}
\]
Your Turn

13. The netting of an empty hammock hangs between its supports along a curve that can be modeled by a parabola. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the curve is point C. Find a quadratic function that models the hammock's netting, and state the function's domain.

\[
g(x) = A f(x-h) + k
\]

\[
g(x) = A f(x-3) + 3
\]

\[
g(5) = A f(5-3) + 3 = 4 = A f(2) + 3
\]

\[
f(2) = \frac{4}{3}
\]

\[
n = A (\frac{4}{3} - \frac{3}{3}) = A (\frac{1}{3})
\]

\[
A = \frac{1}{25} [-2, 8]
\]

Elaborate

14. What is the general procedure to follow when graphing a function of the form \( g(x) = a \cdot f(x-h) + k \) given the graph of \( f(x) \)?

15. What are the general steps to follow when determining the values of the parameters \( a, h, \) and \( k \) in \( f(x) = a(x-h)^2 + k \) when modeling a parabolic real-world object?

16. Essential Question Check-In How can the graph of a function \( f(x) \) be transformed?
Evaluate: Homework and Practice

Write \( g(x) \) in terms of \( f(x) \) after performing the given transformation of the graph of \( f(x) \).

1. Translate the graph of \( f(x) \) to the left 3 units.
   \[
   g(x) = f(x + 3)
   \]
   \[
   g(-3) = f(-3 + 3) = f(0)
   \]

2. Translate the graph of \( f(x) \) up 2 units.
   \[
   g(x) = f(x) + 2
   \]

3. Translate the graph of \( f(x) \) to the right 4 units.
   \[
   g(x) = f(x - 4)
   \]

4. Translate the graph of \( f(x) \) down 3 units.
   \[
   g(x) = f(x) - 3
   \]

5. Stretch the graph of \( f(x) \) horizontally by a factor of 3.

6. Stretch the graph of \( f(x) \) vertically by a factor of 2.
7. Compress the graph of $f(x)$ horizontally by a factor of $\frac{1}{3}$.

8. Compress the graph of $f(x)$ vertically by a factor of $\frac{1}{2}$.

9. Reflect the graph of $f(x)$ across the $y$-axis.

10. Reflect the graph of $f(x)$ across the $x$-axis.

11. Reflect the graph of $f(x)$ across the $y$-axis.

12. Reflect the graph of $f(x)$ across the $x$-axis.
13. Determine if each function is an even function, an odd function, or neither.

\[ a. \ f(x) = 5x^2 \quad \text{ODD} \]
\[ b. \ f(x) = (x - 2)^2 \quad \text{EVEN} \]
\[ c. \ f(x) = 5 \quad \text{NEITHER} \]

14. Determine whether each quadratic function is an even function. Answer yes or no.

\[ a. \ f(x) = \left( \frac{1}{2} \right)^2 \quad \text{YES} \]
\[ b. \ f(x) = (x - 2)(x - 2) \quad \text{YES} \]
\[ c. \ f(x) = x^2 - 2x - 2x + 4 \quad \text{NO} \]
\[ d. \ f(x) = x + 6 \quad \text{YES} \]

Describe how to transform the graph of \( f(x) = x^2 \) to obtain the graph of the related function \( g(x) \). Then draw the graph of \( g(x) \).

\[ g(x) = -\frac{1}{2} f(x + 4) \]
\[ g(x) = \frac{1}{2} f(2x) + 2 \]
17. **Architecture** Flying buttresses were used in the construction of cathedrals and other large stone buildings before the advent of more modern construction materials to prevent the walls of large, high-ceilinged rooms from collapsing.

The design of a flying buttress includes an arch. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the arch is point C. Find a quadratic function that models the arch, and state the function's domain.

\[ g(x) = A(x-h)^2 + k \]

\[ C(2, 12) \]

\[ g(8) = A(8-2)^2 + 12 \]

\[ \ell_0 = A(8-2) + 12 \]

\[ \ell_0 = A6 + 12 \]

\[ \ell_0 = 3\ell_0 + 12 \]

\[ -\ell_0 = A3\ell_0 \]

\[ \ell_0 = A \]

\[ g(x) = \frac{1}{6}f(x-2) + 12 \]

18. A red velvet rope hangs between two stanchions and forms a curve that can be modeled by a parabola. In the illustration shown, the unit of measurement for both axes is feet, and the vertex of the curve is point C. Find a quadratic function that models the rope, and state the function's domain.

\[ g(x) = A(x-h)^2 + k \]

\[ 4 = A(1-4)^2 + 3.5 \]

\[ 4 = A6 + 3.5 \]

\[ 4 = A9 + 3.5 \]

\[ .5 = A9 \]

\[ \frac{.5}{9} = A \]

\[ \frac{18}{9} = A \]

\[ g(x) = \frac{1}{6}f(x-4) + 3.5 \]

\[ [1, 7] \]