10.3 Graphing Cube Root Functions

Essential Question: How can you use transformations of the parent cube root function to graph functions of the form $f(x) = a\sqrt[3]{(x-h)} + k$ or $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$?

Resource Locker

Explore Graphing and Analyzing the Parent Cube Root Function

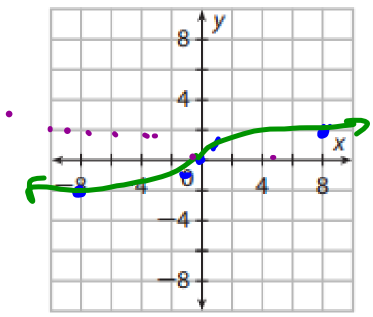
The cube root parent function is $f(x) = \sqrt[3]{x}$. To graph $f(x)$, choose values of x and find corresponding values of y. Choose both negative and positive values of x.

Graph the function $f(x) = \sqrt[3]{x}$. Identify the domain and range of the function.

(A) Make the table of values.

x	y	x, y
−8	-2	
−1	-1	
0	0	
1	1	
8	2	

(B) Use the table to graph the function.



(C) Identify the domain and range of the function.

The domain is the ℝ.

The range is ℝ.

(D) Does the graph of $f(x) = \sqrt[3]{x}$ have any symmetry?

The graph has POINT SYMMETRY

Reflect

1. Can the radicand in a cube root function be negative?

Explain 1 Graphing Cube Root Functions

Transformations of the Cube Root Parent Function $f(x) = \sqrt[3]{x}$

Transformation	f(x) Notation	Examples
Vertical translation	$f(x) + k$	$y = \sqrt[3]{x} + 3$ 3 units up $y = \sqrt[3]{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt[3]{x - 2}$ 2 units right $y = \sqrt[3]{x + 1}$ 1 units left
Vertical stretch/compression	$af(x)$	$y = 6\sqrt[3]{x}$ vertical stretch by a factor of 6 $y = \frac{1}{2}\sqrt[3]{x}$ vertical compression by a factor of $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt[3]{\frac{1}{5}x}$ horizontal stretch by a factor of 5 $y = \sqrt[3]{3x}$ horizontal compression by a factor of $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt[3]{x}$ across x-axis $y = \sqrt[3]{-x}$ across y-axis

For the function $f(x) = a\sqrt[3]{x - h} + k$, (h, k) is the graph's point of symmetry. Use the values of a, h, and k to draw each graph. Note that the point $(1, 1)$ on the graph of the parent function becomes the point $(1 + h, a + k)$ on the graph of the given function.

For the function $f(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$, (h, k) remains the graph's point of symmetry. Note that the point $(1, 1)$ on the graph of the parent function becomes the point $(b + h, 1 + k)$ on the graph of the given function.

Example 1 **Graph the cube root functions.**

(A) Graph $g(x) = 2\sqrt[3]{x - 3} + 5$.

The transformations of the graph of $f(x) = \sqrt[3]{x}$ that produce the graph of $g(x)$ are:

- a vertical stretch by a factor of 2
- a translation of 3 units to the right and 5 units up

Choose points on $f(x) = \sqrt[3]{x}$ and find the transformed corresponding points on $g(x) = 2\sqrt[3]{x - 3} + 5$.

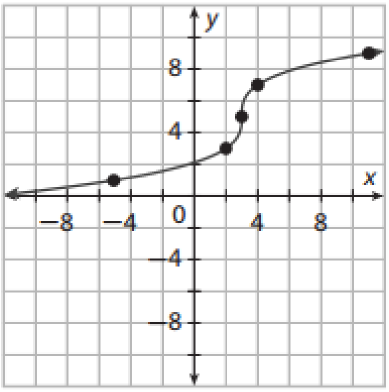
Graph $g(x) = 2\sqrt[3]{x - 3} + 5$ using the transformed points.

x + 3

2 y + 5

x +3
2 y +5

$f(x) = \sqrt[3]{x}$	$g(x) = 2\sqrt[3]{x-3} + 5$
(−8, −2)	(−5, 1)
(−1, −1)	(2, 3)
(0, 0)	(3, 5)
(1, 1)	(4, 7)
(8, 2)	(11, 9)



(B) Graph $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$.

The transformations of the graph of $f(x) = \sqrt[3]{x}$ that produce the graph of $g(x)$ are:

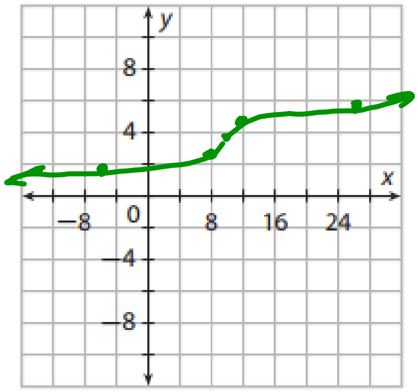
- a horizontal stretch by a factor of 2
- a translation of 10 units to the right and 4 units up

Choose points on $f(x) = \sqrt[3]{x}$ and find the transformed corresponding points on $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$.

Graph $g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$ using the transformed points.

2 x +10   y +4

$f(x) = \sqrt[3]{x}$	$g(x) = \sqrt[3]{\frac{1}{2}(x-10)} + 4$
(−8, −2)	(−6, 2)
(−1, −1)	(8, 3)
(0, 0)	(10, 4)
(1, 1)	(12, 5)
(8, 2)	(26, 6)



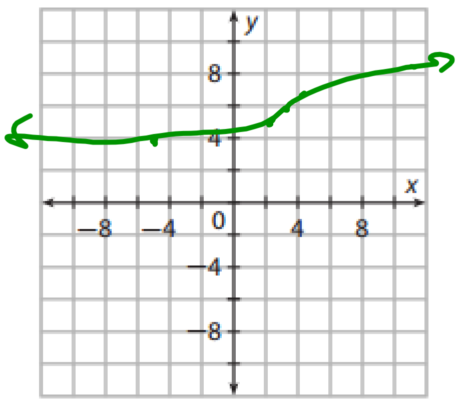
Your Turn

Graph the cube root function.

2. Graph $g(x) = \sqrt[3]{x-3} + 6$.

x +3   y + 6

$f(x) = \sqrt[3]{x}$	$g(x) = \sqrt[3]{x-3} + 6$
(−8, −2)	(−5, 4)
(−1, −1)	(2, 5)
(0, 0)	(3, 6)
(1, 1)	(4, 7)
(8, 2)	(11, 8)

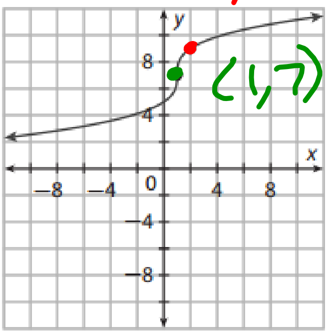


Explain 2 Writing Cube Root Functions

Given the graph of the transformed function $g(x) = a\sqrt[3]{\frac{1}{b}(x-h)} + k$, you can determine the values of the parameters by using the reference points $(-1, 1)$, $(0, 0)$, and $(1, 1)$ that you used to graph $g(x)$ in the previous example.

Example 2 **For the given graphs, write a cube root function.**

(A) Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.



$9 = A\sqrt[3]{2-1} + 7$

$2 = A$

Identify the values of a, h, and k.

Identify the values of h and k from the point of symmetry.

$(h, k) = (1, 7)$, so $h = 1$ and $k = 7$.

Identify the value of a from either of the other two reference points $(-1, 1)$ or $(1, 1)$.

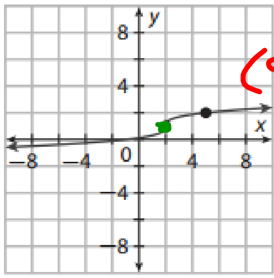
The reference point $(1, 1)$ has general coordinates $(h + 1, a + k)$. Substituting 1 for h and 7 for k and setting the general coordinates equal to the actual coordinates gives this result:

$(h + 1, a + k) = (2, a + 7) = (2, 9)$, so $a = 2$.

$a = 2$ $h = 1$ $k = 7$

The function is $g(x) = 2\sqrt[3]{x-1} + 7$.

(B) Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$.



$2 = \sqrt[3]{\frac{1}{b}(5-2)} + 1$

$1 = \sqrt[3]{\frac{1}{b}(3)}$

$1 = \frac{1}{b}(3)$ $\frac{1}{3} = \frac{1}{b}$

Identify the values of b, h, and k.

Identify the values of h and k from the point of symmetry.

$(h, k) = (2, \boxed{1})$ so $h = 2$ and $k = \boxed{1}$.

Identify the value of b from either of the other two reference points.

The rightmost reference point has general coordinates $(b + h, 1 + k)$. Substituting 2 for h and ____ for k and setting the general coordinates equal to the actual coordinates gives this result:

$(b + h, 1 + \square) = (b + 2, \square) = (5, 2)$, so $b = \square$.

$b = \boxed{3}$ $h = \boxed{2}$ $k = \boxed{1}$

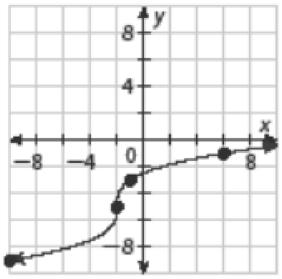
The function is $g(x) = \sqrt[3]{\frac{1}{3}(x-2)} + 1$

Pg 521 2, 7-14. 19-21

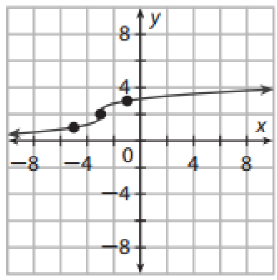
Your Turn

For the given graphs, write a cube root function.

3. Write the function in the form $g(x) = a\sqrt[3]{x - h} + k$.



4. Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$.



Explain 3 Modeling with Cube Root Functions

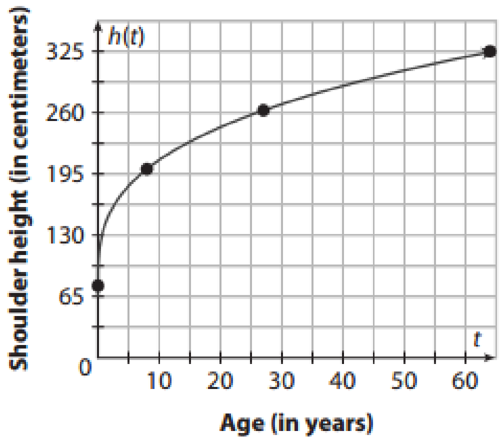
You can use cube root functions to model real-world situations.

Example 3

(A) The shoulder height h (in centimeters) of a particular elephant is modeled by the function $h(t) = 62.1\sqrt[3]{t} + 76$, where t is the age (in years) of the elephant. Graph the function and examine its average rate of change over the equal t-intervals $(0, 20)$, $(20, 40)$, and $(40, 60)$. What is happening to the average rate of change as the t-values of the intervals increase? Use the graph to find the height when $t = 35$.

Graph $h(t) = 62.1\sqrt[3]{t} + 76$.

The graph is the graph of $f(x) = \sqrt[3]{x}$ translated up 76 and stretched vertically by a factor of 62.1. Graph the transformed points $(0, 76)$, $(8, 200.2)$, $(27, 262.3)$, and $(64, 324.4)$. Connect the points with a smooth curve.



First interval:

$$\text{Average Rate of change} \approx \frac{244.6 - 76}{20 - 0}$$
$$= 8.43$$

Second interval:

$$\text{Average Rate of change} \approx \frac{288.4 - 244.6}{40 - 20}$$
$$= 2.19$$

Third interval:

$$\text{Average Rate of change} \approx \frac{319.1 - 288.4}{60 - 40}$$
$$= 1.54$$

The average rate of change is becoming less.

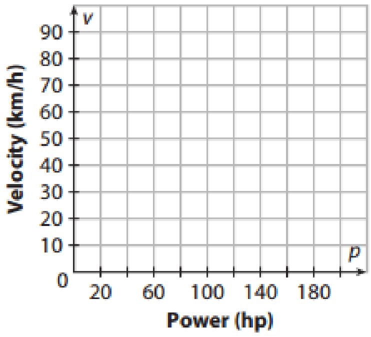
Drawing a vertical line up from 35 gives a value of about 280 cm.

(B) The velocity of a 1400-kilogram car at the end of a 400-meter run is modeled by the function $v = 15.2\sqrt[3]{p}$, where v is the velocity in kilometers per hour and p is the power of its engine in horsepower. Graph the function and examine its average rate of change over the equal p-intervals $(0, 60)$, $(60, 120)$, and $(120, 180)$. What is happening to the average rate of change as the p-values of the intervals increase? Use the function to find the velocity when p is 100 horsepower.

Graph $v = 15.2\sqrt[3]{p}$.

The graph is the graph of $f(x) = \sqrt[3]{x}$ stretched ________ by a factor of 15.2. Graph the transformed points $(0, 0)$, $(8,$ ______$)$, $(27,$ ______$)$, $(64,$ ______$)$, $(125,$ _____$)$, and $(216,$ ______$)$.

Connect the points with a smooth curve.



The average rate of change over the interval $(0, 60)$ is $\dfrac{\square - \square}{60 - 0}$ which is about ______.

The average rate of change over the interval $(60, 120)$ is $\dfrac{\square - \square}{120 - 60}$ whichis about ______.

The average rate of change over the interval $(120, 180)$ is $\dfrac{\square - \square}{180 - 120}$ which is about ______.

The average rate of change is becoming ______.

Substitute $p = 100$ in the function.

$v = 15.2\sqrt[3]{p}$

$v = 15.2\sqrt[3]{\square}$

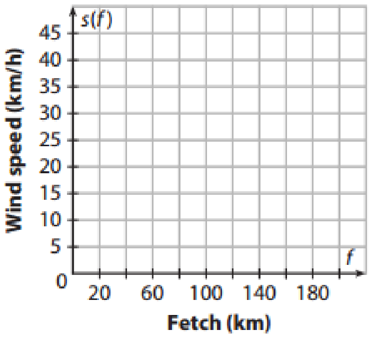
$v \approx 15.2(\square)$

$v \approx \square$

The velocity is about ______km/h.

Your Turn

5. The fetch is the length of water over which the wind is blowing in a certain direction. The function $s(f) = 7.1\sqrt[3]{f}$, relates the speed of the wind s in kilometers per hour to the fetch f in kilometers. Graph the function and examine its average rate of change over the intervals $(20, 80)$, $(80, 140)$, and $(140, 200)$. What is happening to the average rate of change as the f-values of the intervals increase? Use the function to find the speed of the wind when $f = 64$.



Elaborate

6. Discussion Why is the domain of $f(x) = \sqrt[3]{x}$ all real numbers?

7. Identify which transformations (stretches or compressions, reflections, and translations) of $f(x) = \sqrt[3]{x}$ change the following attributes of the function.

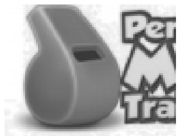
a. Location of the point of symmetry

b. Symmetry about a point

8. Essential Question Check-In How do parameters a, b, h, and k effect the graphs of $f(x) = a\sqrt[3]{(x - h)} + k$ and $g(x) = \sqrt[3]{\frac{1}{b}(x - h)} + k$?

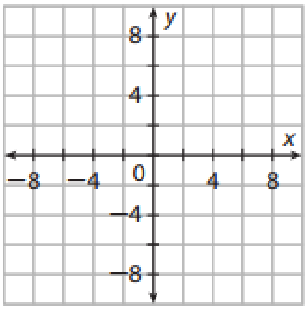
Pg 521 2-6, 9-11, 21

Evaluate: Homework and Practice



- Online
- Hints a
- Extra P

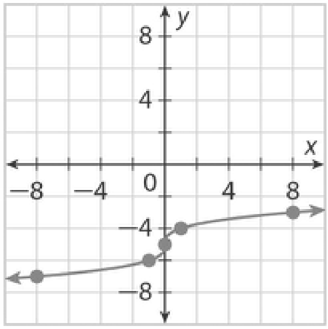
1. Graph the function $g(x) = \sqrt[3]{x} + 3$. Identify the domain and range of the function.



(-8,-2) → -8, -7
(-1,-1) → -1, -6
(0,0) → 0, -5
(1,1) → 1, -4
(8,2) → 8, -3

2. Graph the function $g(x) = \sqrt[3]{x} - 5$. Identify the domain and range of the function.

2.



The domain is all real numbers.

The range is all real numbers.

Describe how the graph of the function compares to the graph of $f(x) = \sqrt[3]{x}$.

3. $g(x) = \sqrt[3]{x} + 6$

3. The graph of $g(x) = \sqrt[3]{x} + 6$ is the graph of $f(x) = \sqrt[3]{x}$ translated 6 units up.

4. $g(x) = \sqrt[3]{x - 5}$

4. The graph of $g(x) = \sqrt[3]{x - 5}$ is the graph of $f(x) = \sqrt[3]{x}$ translated 5 units right.

5. $g(x) = \frac{1}{3}\sqrt[3]{-x}$

5. The graph of $g(x) = \frac{1}{3}\sqrt[3]{-x}$ is the graph of $f(x) = \sqrt[3]{x}$ compressed vertically by a factor of $\frac{1}{3}$ and reflected across the y-axis.

6. $g(x) = \sqrt[3]{5x}$

6. The graph of $g(x) = \sqrt[3]{5x}$ is the graph of $f(x) = \sqrt[3]{x}$ compressed horizontally by a

7. $g(x) = -2\sqrt[3]{x} + 3$

V Stretch 2
Reflection X axis
V. Shift Up 3

8. $g(x) = \sqrt[3]{x + 4} - 3$

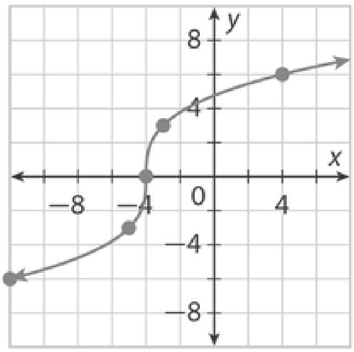
Graph the cube root functions.

9. $g(x) = 3\sqrt[3]{x+4}$

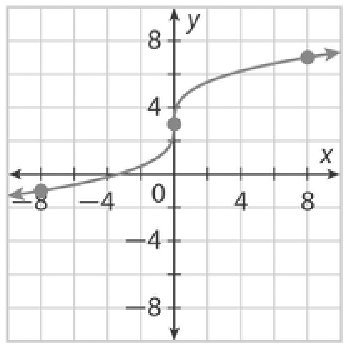
x −4

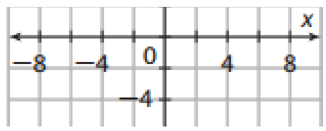
3 y

(−8, −2)(−12, −6)
(−1, −1)(−5, −3)
(0, 0)(−4, 0)
(1, 1)(−3, 3)
(8, 2)(4, 6)

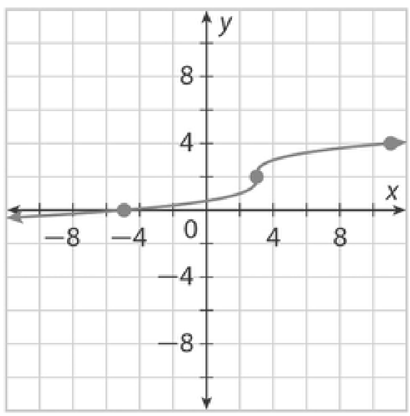


10. $g(x) = 2\sqrt[3]{x} + 3$



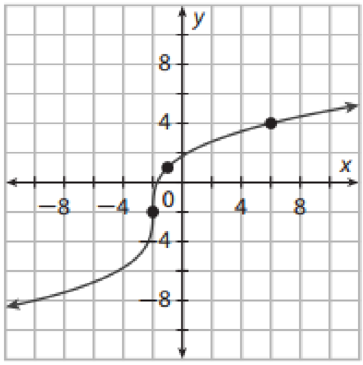


11. $g(x) = \sqrt[3]{x-3} + 2$

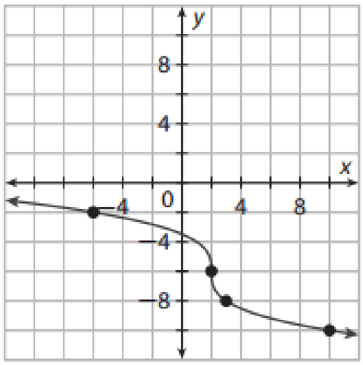


For the given graphs, write a cube root function.

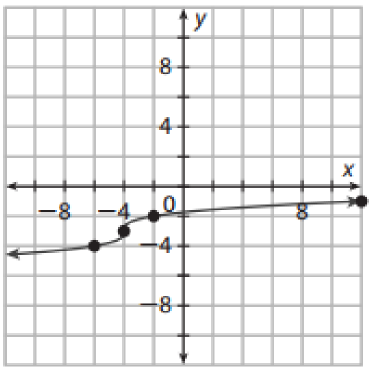
12. Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.



13. Write the function in the form $g(x) = a\sqrt[3]{x-h} + k$.

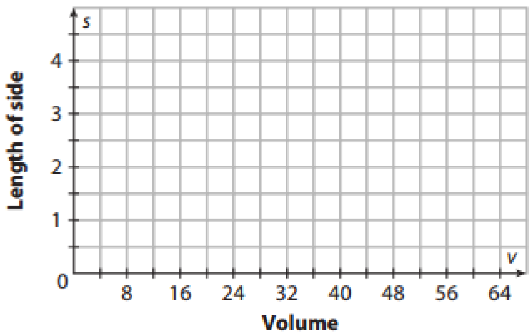


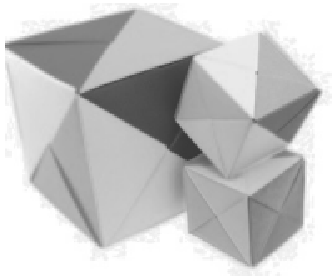
14. Write the function in the form $g(x) = \sqrt[3]{\frac{1}{b}(x-h)} + k$.



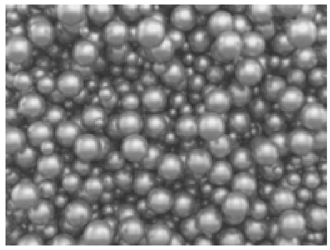
19. Find the y-intercept for the function $y = a\sqrt[3]{x - h} + k$.

3.





16. The radius of a stainless steel ball, in centimeters, can be modeled by $r(m) = 0.31\sqrt[3]{m}$, where m is the mass of the ball in grams. Use the function to find r when $m = 125$.



17. Describe the steps for graphing $g(x) = \sqrt[3]{x + 8} - 11$.

18. **Modeling** Write a situation that can be modeled by a cube root function. Give the function.

24. **Justify Reasoning** Does a horizontal translation and a vertical translation of the function $f(x) = \sqrt[3]{x}$ affect the function's domain or range? Explain.

21. Describe the translation(s) used to get $g(x) = \sqrt[3]{x - 9} + 12$ from $f(x) = \sqrt[3]{x}$. Select all that apply.

A. translated 9 units right

B. translated 9 units left

C. translated 9 units up

D. translated 9 units down

E. translated 12 units right

F. translated 12 units left

G. translated 12 units up

H. translated 12 units down

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** Tim says that to graph $g(x) = \sqrt[3]{x - 6} + 3$, you need to translate the graph of $f(x) = \sqrt[3]{x}$ 6 units to the left and then 3 units up. What mistake did he make?

23. **Communicate Mathematical Ideas** Why does the square root function have a restricted domain but the cube root function does not?

Lesson Performance Task

The side length of a 243-gram copper cube is 3 centimeters. Use this information to write a model for the radius of a copper sphere as a function of its mass. Then, find the radius of a copper sphere with a mass of 50 grams. How would changing the material affect the function?