10.6 Construct and Interpret Binomial Distributions

Goal • Study probability distributions.

ur Notes

VOCABULARY

Random variable A variable whose value is determined by the outcomes of a random event

Probability distribution A function that gives the probability of each possible outcome for a random variable

Binomial distribution A type of probability distribution that shows the probabilities of the outcomes of a binomial experiment

Binomial experiment An experiment that has n independent trials where each outcome has only two possible outcomes: success and failure. The probability for success, p, is the same for each trial.

Symmetric A distribution in which the histogram can be divided into two parts that are mirror images

Skewed A distribution that is not symmetric

PROBABILITY DISTRIBUTIONS

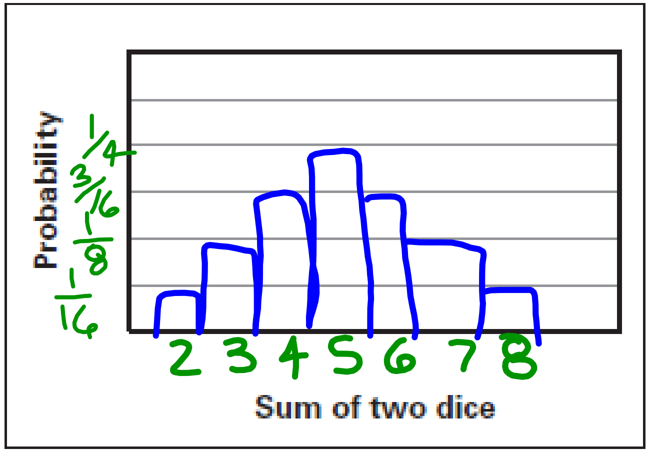
A probability distribution is a function that gives the probability of each possible outcome for a RANDOM VARIABLE. The sum of all the probabilities in a probability distribution must equal 1.

Example 1 *Construct a probability distribution*

Let X be a random variable that represent the sum when two four-sided dice are rolled. Make a table and histogram showing the probability distribution for X.

Solution

The possible values of X are the integers from 2 to 8. The table shows the number of outcomes for each value of X. Divide each value by 16 to get $P(X)$.



X (sum)	2	3	4	5	6	7	8
Outcomes	1	2	3	4	3	2	1
$P(X)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

Example 2 *Interpret a probability distribution*

Use the probability distribution in Example 1 to answer each question. (a) What is the most likely outcome of rolling the two dice? (b) What is the probability that the sum of the two dice is at most 4?

Solution

a. The most likely outcome of rolling the two dice is the value of X for which $P(X)$ is greatest. This probability is greatest for $X =$ 5. So, the most likely outcome when rolling the two dice is a sum of 5.

b. The probability that the sum of the two dice is at most 4 is:

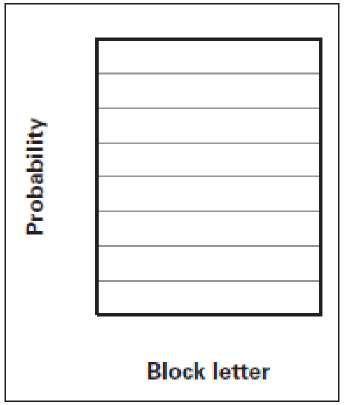
$$P(X \leq 4) = \underline{\qquad\qquad}$$

$$= \frac{1}{16} + \frac{2}{16} + \frac{3}{16} = \frac{3}{8} = .375$$

Checkpoint Complete the following exercise.

1. Let X be the letter on a letter block randomly chosen from a bag containing 7 blocks labeled "A," 3 blocks labeled "B," 6 blocks labeled "C," and 5 blocks labeled "D." Make a table and histogram showing the probability distribution.

X	A	B	C	D
Outcomes	7	3	6	5
$P(X)$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{5}{21}$



BINOMIAL EXPERIMENTS

A binomial experiment meets the following conditions:

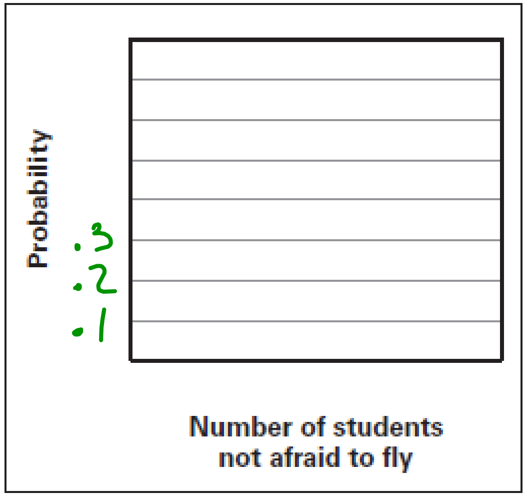
- There are *n* INDEPDENT trials.
- Each trial has only two possible outcomes: Success and FAILUne.
- The probability of success is the SAme for each trial. This probability is denoted by *p*. The probability for failure is given by $1 - p$.

For a binomial experiment, the probability of exactly *k* successes in *n* trials is:

$P(k \text{ successes}) = {}_N C_K P^K (1-P)^{N-K}$

Example 3 *Construct a binomial distribution*

A survey taken in your school found that 68% of the students are not afraid to fly. Suppose you randomly survey 5 students. Draw a histogram of the binomial distribution for your survey.



The probability that a randomly selected student is not afraid to fly is $p =$ .68. Because you survey 5 students, $n =$ 5.

$P(k = 0) = {}_5C_0(.68)^0(.32)^5 \approx .003$

$P(k = 1) = {}_5C_1(.68)^1(.32)^4 \approx .036$

$P(k = 2) = {}_5C_2(.68)^2(.32)^3 \approx .152$

$P(k = 3) = {}_5C_3(.68)^3(.32)^2 \approx .322$

$P(k = 4) = {}_5C_4(.68)^4(.32)^1 \approx .342$

$P(k = 5) = {}_5C_5(.68)^5(.32)^0 \approx .145$

Example 4 ***Interpret a binomial distribution***

Use the binomial distribution in Example 3.

a. What is the most likely outcome of the survey?

b. What is the probability that at least 3 students are not afraid to fly?

Solution

a. The most likely outcome of the survey is the value of k for which $P(k)$ is greatest. This probability is greatest for k = 4. So, the most likely outcome is that 4 of the 5 students are not afraid to fly.

b. The probability that at least 3 students are not afraid to fly is:

$$P(k \geq 3) = P(3) + P(4) + P(5)$$
$$= .322 + .342 + .145 = .809$$

So, the probability is about 80.9%.

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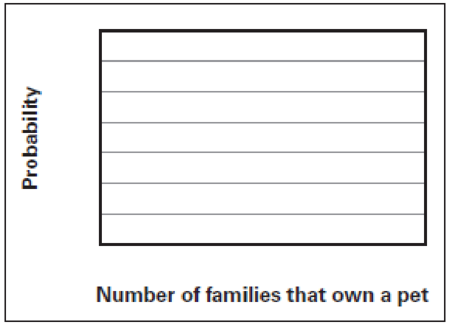
Checkpoint **Complete the following exercises.**

In a survey of your neighborhood, 57% of the families owned a pet. Suppose you randomly survey 6 families.

2. Draw a histogram showing the binomial distribution for your survey.



.0063
.0503
.1666
.2945
.2928
.1552
.0343

3. What is the most likely outcome of your survey? What is the probability that at most 2 families own a pet?