# 1.1 Domain, Range, and End Behavior



**Essential Question:** How can you determine the domain, range, and end behavior of a function?

Resource Locker

### Explore Representing an Interval on a Number Line

An **interval** is a part of a number line without any breaks. A *finite interval* has two endpoints, which may or may not be included in the interval. An *infinite interval* is unbounded at one or both ends.

Suppose an interval consists of all real numbers greater than or equal to 1. You can use the inequality  $x \ge 1$  to represent the interval. You can also use *set notation* and *interval notation*, as shown in the table.

Description of Interval	Type of Interval	Inequality	Set Notation	Interval notation
All real numbers from $a$ to $b$ , including $a$ and $b$	Finite	$a \le x \le b$	$\{x \mid a \leq x \leq b\}$	[a, b]
All real numbers greater than a	Infinite	x > a	$\{x x>a\}$	$(a, +\infty)$
All real numbers less than or equal to <i>a</i>	Infinite	$x \leq a$	$\{x x\leq a\}$	$(-\infty,a]$

For set notation, the vertical bar means "such that," so you read  $\{x | x \ge 1\}$  as "the set of real numbers x such that x is greater than or equal to 1."

For interval notation, do the following:

- Use a square bracket to indicate that an interval includes an endpoint and a parenthesis to indicate
  that an interval doesn't include an endpoint.
- For an interval that is unbounded at its positive end, use the symbol for positive infinity,  $+\infty$ . For an interval that unbounded at its negative end, use the symbol for negative infinity,  $-\infty$ . Always use a parenthesis with positive or negative infinity.

So, you can write the interval  $x \ge 1$  as  $[1, +\infty)$ .

(A) Complete the table by writing the finite interval shown on each number line as an inequality, using set notation, and using interval notation.

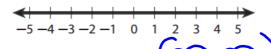
Finite Interval	<del>-5 -4 -3 -2 -1 0 1 2 3 4 5</del>	<del>&lt;++++++++++++++++++++++++++++++++++++</del>	
Inequality		-34 x = Z	
Set Notation		EX1-34X=23	
Interval [-3, 2]		(-3,27)	

B Complete the table by writing the infinite interval shown on each number line as an inequality, using set notation, and using interval notation.

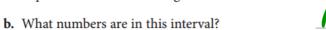
Infinite Interval	<del>-5 -4 -3 -2 -1 0 1 2 3 4 5</del>	<del>&lt;++++++++++++++++++++++++++++++++++++</del>	
Inequality	X <sub>2</sub>	X > 2	
Set Notation	€X X=23	£X   X > 2 }	
Interval Notation	$(-\infty,2)$	(2,00)	

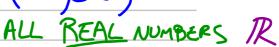
### Reflect

Consider the interval shown on the number line.



a. Represent the interval using interval notation.

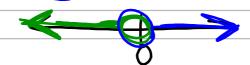






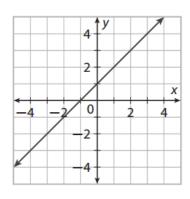
What do the intervals [0, 5], [0, 5), and (0, 5) have in common? What makes them different?

mbol U represents the *union* of two sets. What do you think the notation epresents?



# Explain 1 Identifying a Function's Domain, Range and End Behavior from its Graph

Recall that the *domain* of a function f is the set of input values x, and the *range* is the set of output values f(x). The **end behavior** of a function describes what happens to the f(x)-values as the x-values either increase without bound (approach positive infinity) or decrease without bound (approach negative infinity). For instance, consider the graph of a linear function shown. From the graph, you can make the following observations.



Symbolic Form of Statement
$As x \to +\infty f(x) \to +\infty$
AS X OF (X) OO.

**Example 1** Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function.



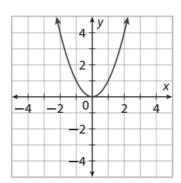
The graph of the quadratic function  $f(x) = x^2$  is shown.

Domain:

Inequality:  $-\infty < x < +\infty$ 

Set notation:  $\{x \mid -\infty < x < +\infty\}$ 

Interval notation:  $(-\infty, +\infty)$ 



Range:

End behavior:

Inequality:  $y \ge 0$ 

As  $x \to +\infty$ ,  $f(x) \to +\infty$ .

Set notation:  $\{y|y \ge 0\}$ 

As  $x \to -\infty$ ,  $f(x) \to +\infty$ .

Interval notation:  $[0, +\infty)$ 

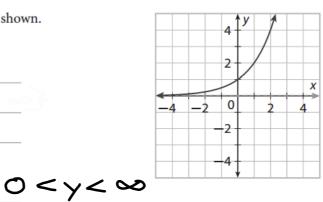
The graph of the exponential function  $f(x) = 2^x$  is shown.

Domain:

Inequality:

Set notation:

Interval notation: \_



Range:

Inequality:

Set notation:

Interval notation: \_

End behavior:

As 
$$r \to +\infty$$

### Reflect

**4.** Why is the end behavior of a quadratic function different from the end behavior of a linear function?

**5.** In Part B, the f(x)-values decrease as the x-values decrease. So, why can't you say that  $f(x) \to -\infty$  as  $x \to -\infty$ ?

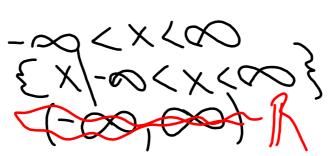
#### **Your Turn**

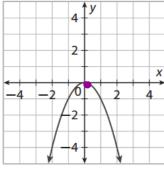
Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function.

**6.** The graph of the quadratic function  $f(x) = -x^2$  is shown.

DOMAIN:

#: S:





RANGE:

END BEHAVIOR:

Τ:

3: EVIYEC

 $\begin{cases} As x \rightarrow -\infty, \beta(x) \end{cases}$ 

IN: (-∞,

## Explain 2 Graphing a Linear Function on a Restricted Domain

Unless otherwise stated, a function is assumed to have a domain consisting of all real numbers for which the function is defined. Many functions—such as linear, quadratic, and exponential functions—are defined all real numbers, so their domain, when written in interval notation,

is  $(-\infty, +\infty)$ . Another way to write the set of real numbers is  $\mathbb{R}$ .

Sometimes a function may have a restricted domain. If the rule for a function and its restricted domain are given, you can draw its graph and then identify its range.

**Example 2** For the given function and domain, draw the graph and identify the range using the same notation as the given domain.

$$A f(x) = \frac{3}{4}x + 2 \text{ with domain} -4 4$$

Since  $f(x) = \frac{3}{4}x + 2$  is a linear function, the graph is a line segment with endpoints at (-4, f(-4)), or (-4, -1), and (4, f(4)), or (4, 5). The endpoints are included in the graph.

The range is 
$$[-1, 5]$$
.

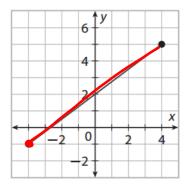
$$5(-4) = \frac{3}{4}(-4) + 2$$

$$5(-4) = -1$$

$$5(4) = \frac{3}{4}(4) + 2$$

$$5(4) = \frac{3}{4}(4) + 2$$

$$5(4) = \frac{3}{4}(4) + 2$$

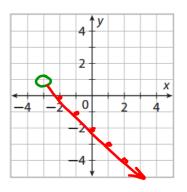


$$f(x) = x - 2 \text{ with domain } \{x | x > -3\}$$

Since f(x) = -x - 2 is a linear function, the graph is a ray with its

endpoint at (-3, f(-3)), or \_\_\_\_\_\_. The endpoint \_\_\_\_\_

included in the graph. The range is \_\_\_\_\_\_.



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### Reflect

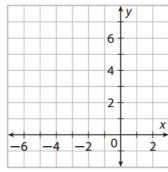
In Part A, how does the graph change if the domain is (-4, 4) instead of [-4, 4]?

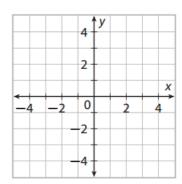
In Part B, what is the end behavior as x increases without bound? Why can't you talk about the end behavior as *x* decreases without bound?

### **Your Turn**

For the given function and domain, draw the graph and identify the range using the same notation as the given domain.

**9.**  $f(x) = -\frac{1}{2}x + 2$  with domain  $-6 \le x < 2$  **10.**  $f(x) = \frac{2}{3}x - 1$  with domain  $(-\infty, 3]$ 





$$\delta(z) = \frac{1}{2}(z) + 2$$

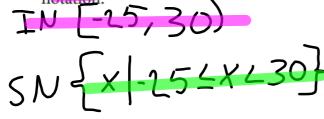


### **Evaluate: Homework and Practice**

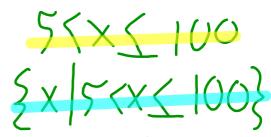
 Write the interval shown on the number line as an inequality, using set notation, and using interval notation.



3. Write the interval  $-25 \le x < 30$  using set notation and interval



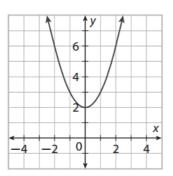
**2.** Write the interval (5, 100] as an inequality and using set notation.



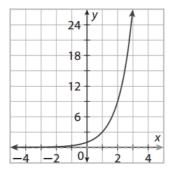
**4.** Write the interval  $\{x \mid -3 < x < 5\}$  as an inequality and using interval notation.

Write the domain and the range of the function as an inequality, using set notation, and using interval notation. Also describe the end behavior of the function or explain why there is no end behavior.

The graph of the quadratic function  $f(x) = x^2 + 2$  is shown.



The graph of the exponential function  $f(x) = 3^x$  is shown.



I: - &< x< &

SN; {X1-00<x<00}

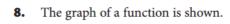
FN: (-∞,∞) /R

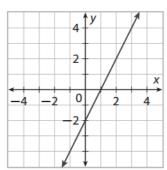
I: 2 < y < 00 }

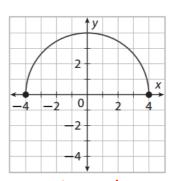
FM: [2,00)

ASX > 0,6(x) > 0

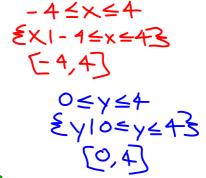
**7.** The graph of the linear function g(x) = 2x - 2 is shown.







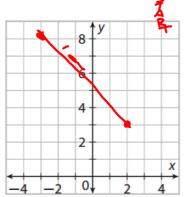
- 60 < X < 60



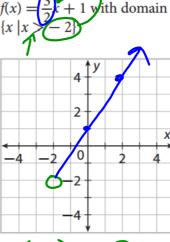
 $ASX \rightarrow \infty, B(X) \rightarrow D$   $ASX \rightarrow \infty, B(X) \rightarrow \infty$ 

For the given function and domain, draw the graph and identify the range using the same notation as the given domain.

**9.** f(x) = -x + 5 with domain [-3, 2]



10.



8(-3)=8

B(2)=3

[3,8]

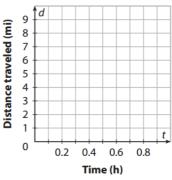
6(-2)=-2

Ey1-2<4<00}

Write a function that models the given situation. Determine the domain from the situation, graph the function using that domain, and identify the range.

**11.** A bicyclist travels at a constant speed of 12 miles per hour for a total of 45 minutes. (Use set notation for the domain and range of the function that models this situation.)





**12.** An elevator in a tall building starts at a floor of the building that is 90 meters above the ground. The elevator descends 2 meters every 0.5 second for 6 seconds. (Use an inequality for the domain and range of the function that models this situation.)

