

1.2 Characteristics of Function Graphs

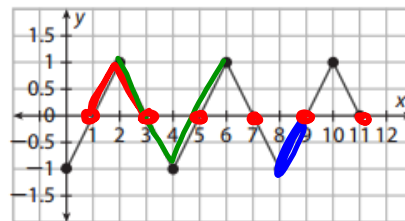


Resource
Locker

Essential Question: What are some of the attributes of a function, and how are they related to the function's graph?

Explore Identifying Attributes of a Function from Its Graph

You can identify several attributes of a function by analyzing its graph. For instance, for the graph shown, you can see that the function's domain is $\{x|0 \leq x \leq 11\}$ and its range is $\{y|-1 \leq y \leq 1\}$. Use the graph to explore the function's other attributes.



- A The values of the function on the interval $\{x|1 < x < 3\}$ are positive/negative.

- B The values of the function on the interval $\{x|8 < x < 9\}$ are positive/negative.

A function is **increasing** on an interval if $f(x_1) < f(x_2)$ when $x_1 < x_2$ for any x -values x_1 and x_2 from the interval. The graph of a function that is increasing on an interval rises from left to right on that interval. Similarly, a function is **decreasing** on an interval if $f(x_1) > f(x_2)$ when $x_1 < x_2$ for any x -values x_1 and x_2 from the interval. The graph of a function that is decreasing on an interval falls from left to right on that interval.

- C The given function is increasing/decreasing on the interval $\{x|2 < x < 4\}$.

- D The given function is increasing/decreasing on the interval $\{x|4 < x < 6\}$.

For the two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ on the graph of a function, the **average rate of change** of the function is the ratio of the change in the function values, $f(x_2) - f(x_1)$, to the change in the x -values, $x_2 - x_1$. For a linear function, the rate of change is constant and represents the slope of the function's graph.

- E What is the given function's average rate of change on the interval $\{x|0 \leq x \leq 2\}$?

Slope = 1

A function may change from increasing to decreasing or from decreasing to increasing at **turning points**. The value of $f(x)$ at a point where a function changes from increasing to decreasing is a **maximum value**. A maximum value occurs at a point that appears higher than all nearby points on the graph of the function. Similarly, the value of $f(x)$ at a point where a function changes from decreasing to increasing is a **minimum value**. A minimum value occurs at a point that appears lower than all nearby points on the graph of the function.

- F At how many points does the given function change from increasing to decreasing? 3



- (G) What is the function's value at these points? 1
- (H) At how many points does the given function change from decreasing to increasing? 2
- (I) What is the function's value at these points? -1

A zero of a function is a value of x for which $f(x) = 0$. On a graph of the function, the zeros are the x -intercepts.

- (J) How many x -intercepts does the given function's graph have? 6
- (K) Identify the zeros of the function. 1, 3, 5, 7, 9, 11
 $x =$

Reflect

1. **Discussion** Identify three different intervals that have the same average rate of change, and state what the rate of change is.
- _____
- _____
- _____
2. **Discussion** If a function is increasing on an interval $\{x|a \leq x \leq b\}$, what can you say about its average rate of change on the interval? Explain.
- _____
- _____
- _____

Explain 1 **Sketching a Function's Graph from a Verbal Description**

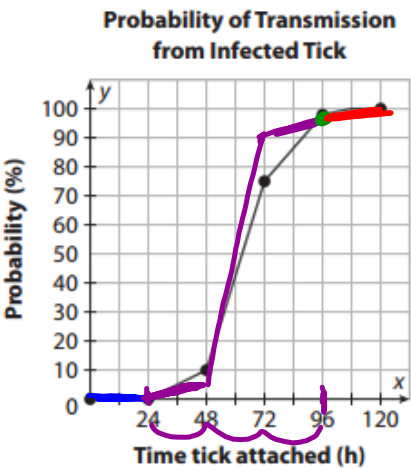
By understanding the attributes of a function, you can sketch a graph from a verbal description.

Example 1 Sketch a graph of the following verbal descriptions.

A Lyme disease is a bacterial infection transmitted to humans by ticks. When an infected tick bites a human, the probability of transmission is a function of the time since the tick attached itself to the skin. During the first 24 hours, the probability is 0%. During the next three 24-hour periods, the rate of change in the probability is always positive, but it is much greater for the middle period than the other two periods. After 96 hours, the probability is almost 100%. Sketch a graph of the function for the probability of transmission.

Identify the axes and scales.

The x -axis will be time (in hours) and will run from 0 to at least 96. The y -axis will be the probability of infection (as a percent) from 0 to 100.



© Houghton Mifflin Harcourt Publishing Company

Identify key intervals.

The intervals are in increments of 24 hours: 0 to 24, 24 to 48, 48 to 72, 72 to 96, and 96 to 120.

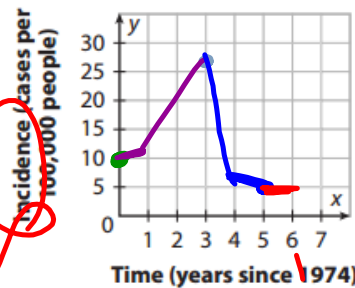
Sketch the graph of the function.

Draw a horizontal segment at $y = 0$ for the first 24-hour interval. The function increases over the next three 24-hour intervals with the middle interval having the greatest increase (the steepest slope). After 96 hours, the graph is nearly horizontal at 100%.

- (B) The incidence of a disease is the rate at which a disease occurs in a population. It is calculated by dividing the number of new cases of a disease in a given time period (typically a year) by the size of the population. **To avoid small decimal numbers, the rate is often expressed in terms of a large number of people rather than a single person.** For instance, the incidence of measles in the United States in 1974 was about 10 cases per 100,000 people.

From 1974 to 1980, there were drastic fluctuations in the incidence of measles in the United States. In 1975, there was a slight increase in incidence from 1974. The next two years saw a substantial increase in the incidence, which reached a maximum in 1977 of about 26 cases per 100,000 people. From 1977 to 1979, the incidence fell to about 5 cases per 100,000 people. The incidence fell much faster from 1977 to 1978 than from 1978 to 1979. Finally, from 1979 to 1980, the incidence stayed about the same. Sketch a graph of the function for the incidence of measles.

Incidence of Measles in the U.S.

**Identify the axes and scales.**

The x-axis will represent time given by years and will run from

0 to 6. The y-axis will

represent _____, measured in cases per 100,000 people, and will run from 0 to 30.

Identify key intervals.

The intervals are one-year increments from 1974 to 1980.

Sketch the graph of the function.

The first point on the graph is (0, 10). The graph slightly rises/falls from $x = 0$ to $x = 1$.

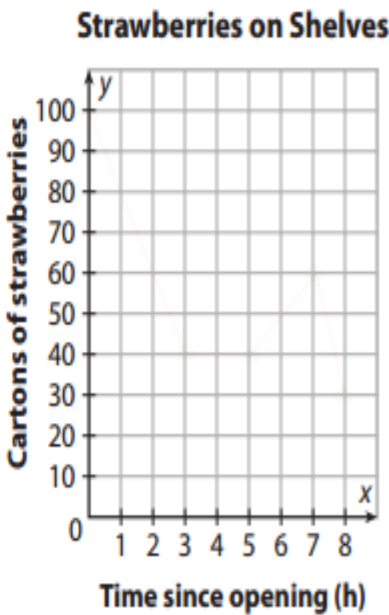
From $x = 1$ to $x = 3$, the graph rises/falls to a maximum y-value of _____. The graph rises/falls steeply from $x = 3$ to $x = 4$ and then rises/falls less steeply from $x = 4$ to $x = 5$. The graph is horizontal from $x = 5$ to $x = 6$.

Reflect

3. In Part B, the graph is horizontal from 1979 to 1980. What can you say about the rate of change for the function on this interval?

Your Turn

4. A grocery store stocks shelves with 100 cartons of strawberries before the store opens. For the first 3 hours the store is open, the store sells 20 cartons per hour. Over the next 2 hours, no cartons of strawberries are sold. The store then restocks 10 cartons each hour for the next 2 hours. In the final hour that the store is open, 30 cartons are sold. Sketch a graph of the function.



Explain 2 Modeling with a Linear Function

When given a set of paired data, you can use a scatter plot to see whether the data show a linear trend. If so, you can use a graphing calculator to perform linear regression and obtain a linear function that models the data. You should treat the least and greatest x -values of the data as the boundaries of the domain of the linear model.

When you perform linear regression, a graphing calculator will report the value of the *correlation coefficient* r . This variable can have a value from -1 to 1 . It measures the direction and strength of the relationship between the variables x and y . If the value of r is negative, the y -values tend to decrease as the x -values increase. If the value of r is positive, the y -values tend to increase as the x -values increase. The more linear the relationship between x and y is, the closer the value of r is to -1 or 1 (or the closer the value of r^2 is to 1).

You can use the linear model to make predictions and decisions based on the data. Making a prediction within the domain of the linear model is called *interpolation*. Making a prediction outside the domain is called *extrapolation*.

Example 2 Perform a linear regression for the given situation and make predictions.

- A** A photographer hiked through the Grand Canyon. Each day she stored photos on a memory card for her digital camera. When she returned from the trip, she deleted some photos from each memory card, saving only the best. The table shows the number of photos she kept from all those stored on each memory card. Use a graphing calculator to create a scatter plot of the data, find a linear regression model, and graph the model. Then use the model to predict the number of photos she will keep if she takes 150 photos.

Grand Canyon Photos	
Photos Taken	Photos Kept
117	25
128	31
140	39
157	52
110	21
188	45
170	42

NORMAL FLOAT AUTO REAL RADIAN MP

LinReg

$y = ax + b$

$a = .33097043$

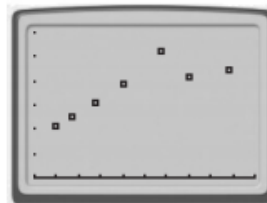
$b = -11.32573346$

$$Y = .331X - 11.326$$

Step 1: Create a scatter plot of the data.

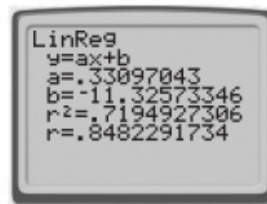
Let x represent the number of photos taken, and let y represent the number of photos kept. Use a viewing window that shows x -values from 100 to 200 and y -values from 0 to 60.

Notice that the trend in the data appears to be roughly linear, with y -values generally increasing as x -values increase.



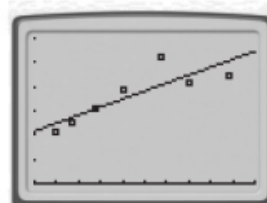
Step 2: Perform linear regression. Write the linear model and its domain.

The linear regression model is $y = 0.33x - 11.33$. Its domain is $\{x | 110 \leq x \leq 188\}$.



Step 3: Graph the model along with the data to obtain a visual check on the goodness of fit.

Notice that one of the data points is much farther from the line than the other data points are. The value of the correlation coefficient r would be closer to 1 without this data point.



Step 4: Predict the number of photos this photographer will keep if she takes 150 photos.

Evaluate the linear function when $x = 150$: $y = 0.33(150) - 11.33 \approx 38$. So, she will keep about 38 photos if she takes 150 photos.

- B As a science project, Shelley is studying the relationship of car mileage (in miles per gallon) and speed (in miles per hour). The table shows the data Shelley gathered using her family’s vehicle. Use a graphing calculator to create a scatter plot of the data, find a linear regression model, and graph the model. Then use the model to predict the gas mileage of the car at a speed of 20 miles per hour.

Speed (mi/h)	30	40	50	60	70
Mileage (mi/gal)	34.0	33.5	31.5	29.0	27.5

Step 1: Create a scatter plot of the data.

What do x and y represent?

X is speed

Y is Mileage

What viewing window will you use?

What trend do you observe?

It looks like a linear function with a negative correlation.

NORMAL FLOAT AUTO REAL RADIAN MP

LinReg

$$y = ax + b$$
$$a = -.175$$
$$b = 39.85$$

$$y = -.175x + 39.85$$

Step 2: Perform linear regression. Write the linear model

Step 3: Graph the model along with the data to obtain a visual check on the goodness of fit.

What can you say about the goodness of fit?

Step 4: Predict the gas mileage of the car at a speed of 20 miles per hour.

Reflect

5. Identify whether each prediction in Parts A and B is an interpolation or an extrapolation.

Your Turn

6. Vern created a website for his school's sports teams. He has a hit counter on his site that lets him know how many people have visited the site. The table shows the number of hits the site received each day for the first two weeks. Use a graphing calculator to find the linear regression model. Then predict how many hits there will be on day 15.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hits	5	10	21	24	28	36	33	21	27	40	46	50	31	38

Elaborate

7. How are the attributes of increasing and decreasing related to average rate of change? How are the attributes of maximum and minimum values related to the attributes of increasing and decreasing?

8. How can line segments be used to sketch graphs of functions that model real-world situations?

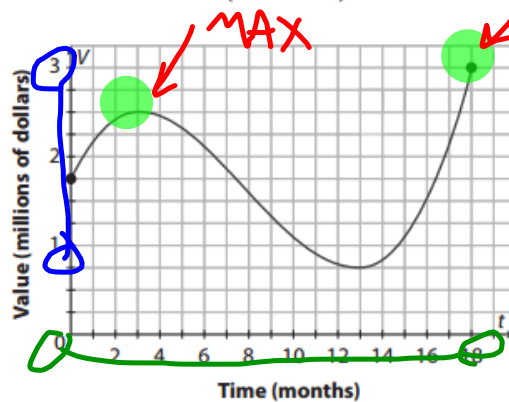
9. When making predictions based on a linear model, would you expect interpolated or extrapolated values to be more accurate? Justify your answer.

10. Essential Question Check-In What are some of the attributes of a function?



Evaluate: Homework and Practice

The graph shows a function that models the value V (in millions of dollars) of a stock portfolio as a function of time t (in months) over an 18-month period.



Pg 24

7(A-D) 8-12, 17
FIRST

1. On what interval is the function decreasing?

1. $\{t | 3 \leq t \leq 13\}$

$[3, 13]$

On what intervals is the function increasing? $\{t | 0 \leq t \leq 3\}$ and $\{t | 13 \leq t \leq 18\}$

2. Identify any maximum values and minimum values.

2. A maximum value of 2.5 occurs at $x = 3$. A minimum value of 0.75 occurs at $x = 13$.

3. What are the function's domain and range?

3. Domain: $\{t | 0 \leq t \leq 18\}$; Range:

$[0, 18]$

$\{V(t) | 0.75 \leq V(t) \leq 3\}$.

The table of values gives the probability $P(n)$ for getting all 5's when rolling a number cube n times.

n	1	2	3	4	5
$P(n)$	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{216}$	$\frac{1}{1296}$	$\frac{1}{7776}$

4. Is $P(n)$ increasing or decreasing? Explain the significance of this.

6. Choices A, C, and F all have positive average rates of change.

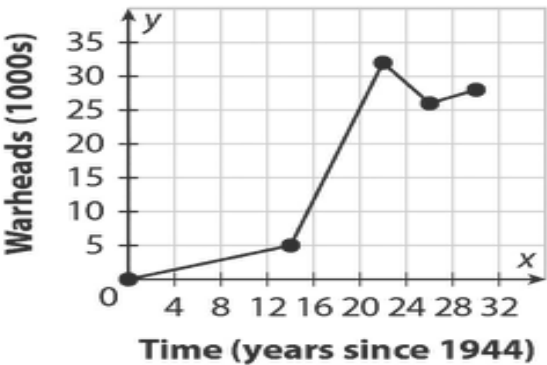
7. 2
-2
-1
4
-3
0

8. $\{x|-3 < x < -1\}$ and $\{x|x > 2\}$.

9. $\{x|x < -3\}$ and $\{x|-1 < x < 2\}$.

10. -3, -1, and 2

11.



6. The table shows some values of a function. On which intervals is the function's average rate of change positive? Select all that apply.

x	0	1	2	3
$f(x)$	50	75	40	65

- a. From $x = 0$ to $x = 1$ c. From $x = 0$ to $x = 3$ e. From $x = 1$ to $x = 3$
 b. From $x = 0$ to $x = 2$ d. From $x = 1$ to $x = 2$ f. From $x = 2$ to $x = 3$

Pg 25 7-12, (17,18 regression equation only)

Use the graph of the function $f(x)$ to identify the function's specified attributes.

7. Find the function's average rate of change over each interval.

a. From $x = -3$ to $x = -2$

2

b. From $x = -2$ to $x = 1$

$$\frac{2 - (-4)}{-2 - 1} = -2$$

c. From $x = 0$ to $x = 1$

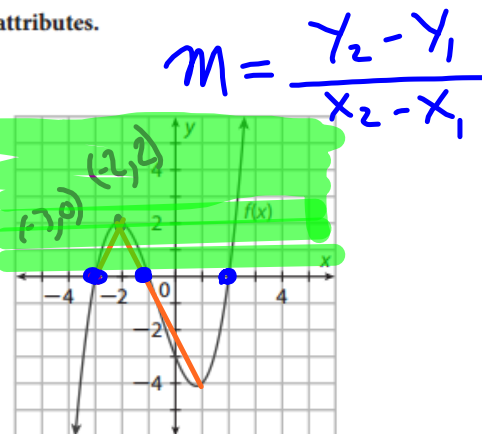
$$\frac{-3 - (-4)}{0 - 1} = \frac{1}{-1} = -1$$

e. From $x = -1$ to $x = 0$

d. From $x = 1$ to $x = 2$

$$\frac{0 - (-4)}{2 - 1} = \frac{4}{1} = 4$$

f. From $x = -1$ to $x = 2$



8. On what intervals are the function's values positive? 8. $\{x | -3 < x < -1\}$ and $\{x | x > 2\}$.

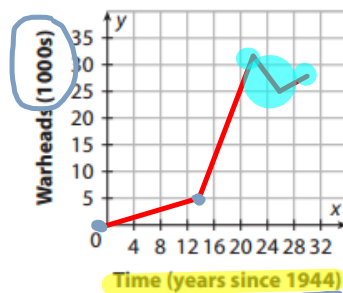
$(-3, -1) \cup (2, \infty)$

9. On what intervals are the function's values negative? 9. $\{x | x < -3\}$ and $\{x | -1 < x < 2\}$.

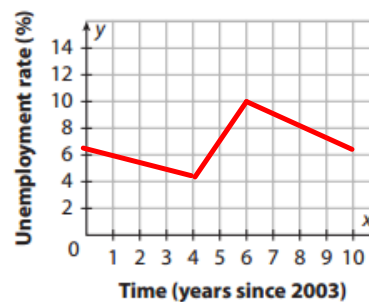
10. What are the zeros of the function? 10. -3, -1, and 2.

\rightarrow X INTERCEPTS

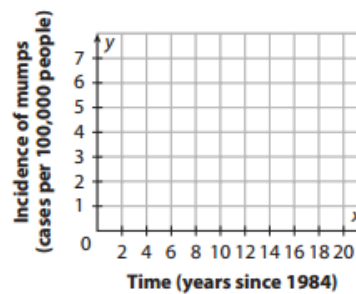
11. The following describes the United States nuclear stockpile from 1944 to 1974. From 1944 to 1958, there was a gradual increase in the number of warheads from 0 to about 5000. From 1958 to 1966, there was a rapid increase in the number of warheads to a maximum of about 32,000. From 1966 to 1970, there was a decrease in the number of warheads to about 26,000. Finally, from 1970 to 1974, there was a small increase to about 28,000 warheads. Sketch a graph of the function.



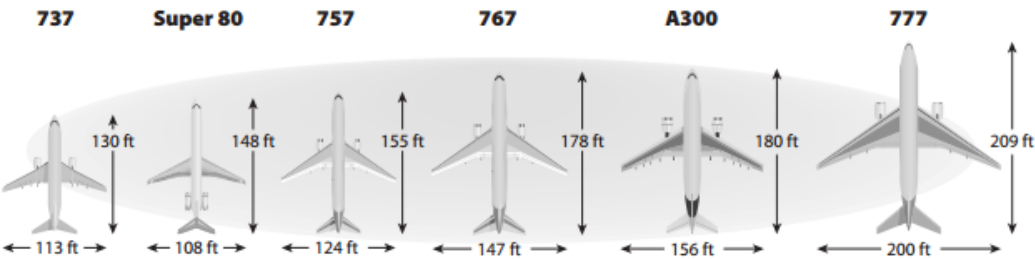
12. The following describes the unemployment rate in the United States from 2003 to 2013. In 2003, the unemployment rate was at 6.3%. The unemployment rate began to fall over the years and reached a minimum of about 4.4% in 2007. A recession that began in 2007 caused the unemployment rate to increase over a two-year period and reach a maximum of about 10% in 2009. The unemployment rate then decreased over the next four years to about 7.0% in 2013. Sketch a graph of the function.



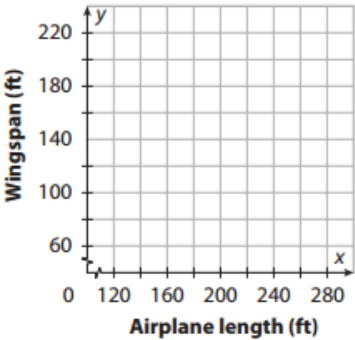
13. The following describes the incidence of mumps in the United States from 1984 to 2004. From 1984 to 1985, there was no change in the incidence of mumps, staying at about 1 case per 100,000 people. Then there was a spike in the incidence of mumps, which reached a peak of about 5.5 cases per 100,000 in 1987. Over the next year, there was a sharp decline in the incidence of mumps, to about 2 cases per 100,000 people in 1988. Then, from 1988 to 1989, there was a small increase to about 2.5 cases per 100,000 people. This was followed by a gradual decline, which reached a minimum of about 0.1 case per 100,000 in 1999. For the next five years, there was no change in the incidence of mumps. Sketch a graph of the function.



14. Aviation The table gives the lengths and wingspans of airplanes in an airline's fleet.



- a. Make a scatter plot of the data with x representing length and y representing wingspan.
- b. Sketch a line of fit.
- c. Use the line of fit to predict the wingspan of an airplane with a length of 220 feet.



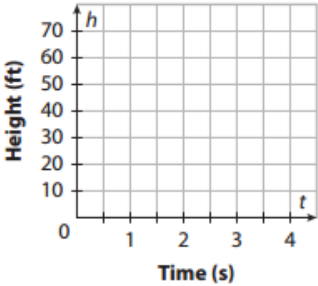
© Houghton Mifflin Harcourt Publishing Company

15. **Golf** The table shows the height (in feet) of a golf ball at various times (in seconds) after a golfer hits the ball into the air.

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
Height (ft)	0	28	48	60	64	60	48	28	0



- a. Graph the data in the table. Then draw a smooth curve through the data points. (Because the golf ball is a projectile, its height h at time t can be modeled by a quadratic function whose graph is a parabola.)
- b. What is the maximum height that the golf ball reaches?
- c. On what interval is the golf ball's height increasing?
- d. On what interval is the golf ball's height decreasing?



16. The model $a = 0.25t + 29$ represents the median age a of females in the United States as a function of time t (in years since 1970).
- a. Predict the median age of females in 1995.
 - b. Predict the median age of females in 2015 to the nearest tenth.



H.O.T. Focus on Higher Order Thinking

- 17. Make a Prediction** Anthropologists who study skeletal remains can predict a woman's height just from the length of her humerus, the bone between the elbow and the shoulder. The table gives data for humerus length and overall height for various women.



Humerus Length (cm)	35	27	30	33	25	39	27	31
Height (cm)	167	146	154	165	140	180	149	155

Using a graphing calculator, find the linear regression model and state its domain. Then predict a woman's height from a humerus that is 32 cm long, and tell whether the prediction is an interpolation or an extrapolation.

NORMAL FLOAT AUTO REAL Radian MP

LinReg

$y=ax+b$
 $a=2.753883892$
 $b=71.97383483$

$$Y = \underline{2.754}X + \underline{71.974}$$

- 18. Make a Prediction** Hummingbird wing beat rates are much higher than those in other birds. The table gives data about the mass and the frequency of wing beats for various species of hummingbirds.



Mass (g)	3.1	2.0	3.2	4.0	3.7	1.9	4.5
Frequency of Wing Beats (beats per second)	60	85	50	45	55	90	40

NORMAL FLOAT AUTO REAL Radian MP

LinReg

$y=ax+b$
 $a=-19.14335664$
 $b=121.973027$

- a. Using a graphing calculator, find the linear regression model and state its domain.

- b. Predict the frequency of wing beats for a Giant Hummingbird with a mass of 19 grams.

$$Y = -19.143X + 121.973$$

- c. Comment on the reasonableness of the prediction and what, if anything, is wrong with the model.