

Busch

## **Algebra II**

Algebra II is a continuation of the mastery of the standards of calculating using algebraic rules introduced in Algebra 1. You will also learn to connect mathematical concepts, reasoning, problem solving, and communicating of mathematical concepts.

Algebra II includes the study of quadratic and polynomial functions, logarithmic and exponential functions, trigonometry, systems of linear equations, sequences and series, probability and analytic geometry. This course will cover the scientific applications of the above topics in order to prepare you for Precalculus.

### **1<sup>st</sup> 9 Weeks:**

- 1) Analyzing Functions
  - A) Domain, Range, End Behavior
  - B) Characteristics of Function Graphs
  - C) Transformations of Function Graphs
  - D) Inverses of Functions
- 2) Absolute Value Functions, Equations, and Inequalities
  - A) Graphing Absolute Value Functions
  - B) Solving Absolute Value Equations
  - C) Solving Absolute Value Inequalities
- 3) Quadratic Equations
  - A) Solving Quadratic Equations by taking Square Roots
  - B) Complex Numbers
  - C) Finding Complex Solutions of Quadratic Equations

### **2<sup>nd</sup> 9 Weeks:**

- 1) Quadratic Relations and Systems of Equations
  - A) Circles
  - B) Parabolas
  - C) Solving Linear-Quadratic Systems
  - D) Solving Linear Systems in Three Variables
- 2) Polynomial Functions
  - A) Graphing Cubic Functions
  - B) Graphing Polynomial Functions
- 3) Polynomials
  - A) Adding and Subtracting Polynomials
  - B) Multiplying Polynomials
  - C) The Binomial Theorem
  - D) Factoring Polynomials
  - E) Dividing Polynomials

**3<sup>rd</sup> 9 Weeks:**

- 1) Polynomial Equations
  - A) Finding Rational Solutions
  - B) Finding Complex Solutions
- 2) SAT preparation
- 3) Rational Functions
  - A) Graphing Simple Rational Functions
  - B) Graphing More Complicated Rational Functions
- 4) Rational Expressions and Equations
  - A) Adding and Subtracting Rational Expressions
  - B) Multiplying and Dividing Rational Expressions
  - C) Solving Rational Equations

**4<sup>th</sup> 9 Weeks:**

- 1) Radical Functions
  - A) Inverses of Simple Quadratic and Cubic Functions
  - B) Graphing Square Root Functions
  - C) Graphing Cube Root Functions
- 2) Unit Circle Definition of Trigonometric Functions
  - A) Angles of Rotation and Radian Measure
  - B) Defining and Evaluating the Basic Trigonometric Functions
  - C) Using a Pythagorean Identity
- 3) Graphing Trigonometric Functions
  - A) Stretching Compressing, and Reflecting Sine and Cosine Graphs
  - B) Stretching, Compressing, and Reflecting Tangent Graphs
  - C) Translating Trigonometric Graphs
  - D) Fitting Sine Functions to Data

## LCCC College Algebra

Review of intermediate algebraic concepts; Solving linear, quadratic, and non-linear equations; Analytic geometry of straight lines and circles; Solving linear and non-linear inequalities; Graphing equations; Concepts of function, domain, range; Function graphing techniques; Inverse functions; Quadratic, polynomial, rational functions; Zeros of polynomials, rational zero theorem; Linear and non-linear systems of equations; Gaussian elimination; Matrices and operations on matrices; Using matrices to solve systems of equations; Exponential and logarithmic functions; Solving exponential and logarithmic equations; and Partial fractions. This is roughly the first six chapters, plus chapter eight in the text.

<i>Class #</i>	<i>Date</i>	<i>Section</i>	<i>Topic</i>
1	Tuesday August 27	Getting Started	Getting Started With Webassign
2	Wednesday August 28	1.3	Introduction to Functions
3	Thursday August 29		
4	Friday August 30	1.4	Function Notation
5	Tuesday September 3	1.5	Function Arithmetic
6	Wednesday September 4		
7	Thursday September 5		
8	Friday September 6	1.6	Graphs of Functions
9	Monday September 9		
10	Tuesday September 10	1.7	Transformations
11	Thursday September 12	1.7	Transformations
12	Friday September 13	2.1	Linear Functions
13	Monday September 16		
14	Tuesday September 17	Exam Review	
15	Thursday September 19	<b>Exam 1</b>	<b>Sections 1.1 – 1.7, 2.1</b>
16	Friday September 20	2.2	Absolute Value Functions
17	Monday September 23		
18	Tuesday September 24	2.3	Quadratic Functions
19	Thursday September 26	2.4	Inequalities
20	Friday September 27		
21	Monday September 30	3.1	Graphs of Polynomials
22	Tuesday October 1		
23	Thursday October 3	3.2	The Factor Theorem and the Remainder Theorem
24	Friday October 4		
25	Monday October 7	3.3	Real Zeros of a Polynomial
26	Tuesday October 8		

27	Thursday October 10	3.4	Complex Zeros of Polynomials
28	Monday October 14		
29	Tuesday October 15	Exam Review	
30	Thursday October 17	<b>Exam 2</b>	<b>Sections 2.2-2.4, 3.1-3.4</b>
31	Friday October 18	4.1	Introduction to Rational Functions
32	Monday October 21		
33	Tuesday October 22	4.2	Graphs of Rational Functions
34	Thursday October 24	4.3	Rational Inequalities and Applications
35	Friday October 25	5.1	Function Composition
36	Monday October 28		
37	Tuesday October 29	5.2	Inverse Functions
38	Thursday October 31	5.3	Other Algebraic Functions
39	Friday November 1		
40	Tuesday November 5	Exam Review	
41	Thursday November 7	<b>Exam 3</b>	<b>Sections 4.1-5.3</b>
42	Friday November 8	6.1	Introduction to Logarithmic and Exponential Functions
43	Monday November 11		
44	Tuesday November 12	6.2	Properties of Logarithms
***	Wednesday November 13	6.3	Exponential Equations and Inequalities
45	Thursday November 14		
46	Friday November 15	6.4	Exponential and Logarithmic Equations and Inequalities
47	Monday November 18		
48	Tuesday November 19	6.4	Logarithmic Equations and Inequalities
***	Wednesday November 20	6.5	Applications of Logarithmic and Exponential Functions
49	Thursday November 21	6.5	Applications of Logarithmic and Exponential Functions
50	Friday November 22	Exam Review	
51	Monday November 25	<b>EXAM 4</b>	<b>Sections 6.1-6.5</b>
52	Tuesday November 26	<b>EXAM 4</b>	<b>Sections 6.1-6.5</b>
53	Monday December 2	8.1	Systems of Linear Equations: Gaussian Elimination
54	Tuesday December 3	8.2	Systems of Linear Equations: Augmented Matrices
55	Thursday December 5	8.3	Matrix Arithmetic
56	Friday December 6		
57	Monday December 9	8.4	Systems of Linear Equations: Matrix Inverses
58	Tuesday December 10		
59	Thursday December 12	Exam Review	
60	Friday December 13	<b>Exam 5</b>	<b>Sections 8.1-8.4</b>
61	Monday December 16	Review for Final	
62	Tuesday December 17	Review for Final	
63	December 18-20	<b>Final Exam</b>	<b>Final Exam</b>

## Advanced Placement Calculus

### Topic Outline:

#### I. **Functions, Graphs, and Limits**

**Analysis of graphs** With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of Calculus both to predict and to explain the observed local and global behavior of a function.

**Limits of Functions (Including one-sided limits)** An intuitive understanding of the limiting process, calculate limits using algebra, and estimating limits from graphs or tables of data. *Students will find limits through substitution, simplification, complexification, as well as using their graphing calculators to explore the limit through the use of tables and graphs.*

**Asymptotic and unbounded behavior** Understanding asymptotes in terms of graphical behavior, describing asymptotic behavior in terms of limits involving infinity, comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

**Continuity as a property of functions** an intuitive understanding of a continuity, understanding continuity in terms of limits, and geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

**L'Hopital's Rule** will be covered later in the year after derivatives are fully understood. A good review activity here is done with partners and cards with functions and another set of cards with the result of L'Hopital's rule. Cards with graphs or end behavior can be entered as well.

#### II. **Derivatives**

**Concept of the derivative.** Derivative presented graphically, numerically, and analytically. Derivatives interpreted as an instantaneous rate of change, defined as the limit of the difference quotient, and the relationship between differentiability and continuity. *Students will work in groups on a variety of different types of functions with the limit of the difference quotient before moving on to discover the rules. All types of differentiability are discussed in exploratory lesson on corners, cusps, vertical tangents and discontinuity. It is critical that  $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$  is understood to be the slope of  $f(x)$  at  $x=a$  before moving on.*

**Derivative at a point.** Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents. Tangent line to a curve at a point and local linear approximation, instantaneous rate of change as the limit of the average rate of change, and approximate rate of change from graphs and tables of values. *Activity: Using the graphing calculator and two closely related functions students will be able to zoom in and see the difference between what happens to a locally linear function versus one with a corner.*

**Derivative as a function.** Corresponding characteristics of the graphs of  $f$  and  $f'$  such as the relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$ . The mean value theorem and its geometric consequences. Equations involving derivatives, verbal descriptions translated into equations and vice versa. *Homework assignments include descriptive comparisons of  $f$ ,  $f'$ , and  $f''$  as well as sketches of all three based off of other graphs and/or charts of information.*

**Second derivatives.** Corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$ . The relationship of concavity of  $f$  and the sign of  $f''$ . Points of inflection and where concavity changes. *A take home test is issued at this time to check understanding of the finer points of all the vocabulary; critical points, local versus global extreme values, points of inflection, increasing/decreasing, and concavity are all solved and graphed with justification sentences provided. Each student has a unique equation and must give a detailed description including all steps and explaining all parts of the graph. Justification sentences must be perfect before moving on.*

**Applications of derivatives.** Analysis of curves, including the notions of monotonicity and concavity. Optimization, both absolute (global), and relative (local) extrema. Use implicit differentiation to find the derivative of an inverse function. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations. *Each student will participate in an activity involving a Vernier gomonion! sensor and a graph from a TI calculator. Students first read position and velocity graphs before recreating the movement themselves in front of the gomonion! Students must demonstrate ability to read and recreate a graph before moving on.*

*Students will also be placed in pairs and given a real life situation to work out a related rate involved equation. Presentations will be made by the team to verify deep understanding during oral presentation.*

*Students will also work in groups to complete Joys of the Intermediate Value Theorem by James Tanton to gain a deeper understanding of the application possibilities of the Intermediate Value Theorem.*

**Computation of derivatives.** Knowledge of derivatives of basic functions, including powers, exponential, logarithmic, trigonometric, and inverse trigonometric functions. Basic rules for the derivatives of sums, products, and quotients of functions, as well as the chain rule and implicit differentiation. *Proofs of all rules will be explored for deeper understanding.*

### III. Integrals

**Interpretation and properties of definite integrals** Definite integral as a limit of Riemann sums, the properties of basic integrals, and the rate of change over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

*During an in class activity students will demonstrate the ability to compute all types of Riemann sums with a varying number of partitions. We focus on one equation over a single set interval and find the left, right, and midpoint Riemann sums multiple times each with increasing partitions. The connection of infinite partitions to the definite integral can now be made. The Fundamental Theorem of Calculus will be added later to further explore connections.*

*Students will explore the transition from a limit as  $n$  goes to infinity with an expression shown as a summation into proper integration notation, and evaluate the function over a closed interval translated through the limits of a function.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n f(c_k)\Delta x$  on the interval  $[a,b]$  is  $\int_a^b f(x)dx$*

**Applications of integrals** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line. *Students will work with a variety of data showing competence of accumulation. This includes data collected on my thirty minute drive to school to see who can properly estimate this accumulating distance.*

**Fundamental Theorem of Calculus** Use the Fundamental Theorem of Calculus to evaluate definite and to represent a particular antiderivative, along with graphical analysis of functions so defined.

**Techniques of antidifferentiation** Antiderivatives following directly from derivatives of basic functions or by substitution of variables (including change of limits for definite integrals) *During substitution with definite integrals notation is stressed as students are shown how problems can be done by substituting the limits early in the problem, or not at all if the antiderivative is substituted back to the original variable.*

**Applications of antidifferentiation** Finding specific antiderivatives using initial conditions, including applications to motion along a line and solving separable equations, and using them in modeling (exponential growth)

**Numerical approximations to definite integrals** Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by a table of values.

#### **IV. AP Review**

We will review practice AP tests. This includes answering multiple choice and extended response questions under the proper time constraints. Students will also use Geogebra combined with Audrey Weeks Calculus in Motion to look at old extended response questions. Students will have an opportunity to do things like create a slider showing a function and its derivative. The graph shows the tangent line sliding along the curve as its slope values are plotted. Polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions are used as the parent function on different days.

#### **Post AP Exam:**

Students will work individually to create their own rational equation and express all the concepts we have learned in one 13 page portfolio. Contained within the pages are detailed descriptions of work necessary to find extreme values, concavity, graphs, mean value theorem, Riemann sums, volume of revolution, average value, and more. All concepts are also expressed through hand drawn graphs. This portfolio has proven to be a valuable asset to any student continuing down the calculus path in need of a review in college.