2.2 Solving Absolute Value Equations

Essential Question: How can you solve an absolute value equation?

Resource Locker

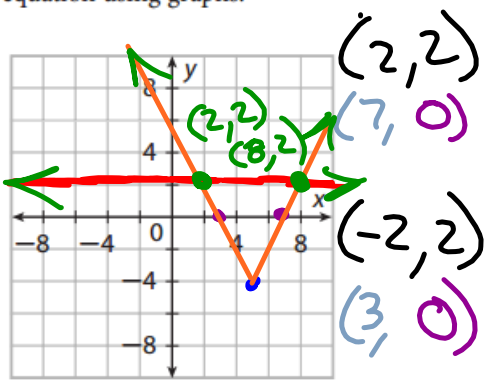
Explore Solving Absolute Value Equations Graphically

Absolute value equations differ from linear equations in that they may have two solutions. This is indicated with a **disjunction**, a mathematical statement created by a connecting two other statements with the word "or." To see why there can be two solutions, you can solve an absolute value equation using graphs.

VERTEX (5, −4)

(A) Solve the equation $2|x - 5| - 4 = 2$.

Plot the function $f(x) = 2|x - 5| - 4$ on the grid. Then plot the function $g(x) = 2$ as a horizontal line on the same grid, and mark the points where the graphs intersect.



(B) Write the solution to this equation as a disjunction:

$x =$ 2 or $x =$ 8

Reflect

1. Why might you expect most absolute value equations to have two solutions? Why not three or four?

2. Is it possible for an absolute value equation to have no solutions? one solution? If so, what would each look like graphically?

Explain 1 Solving Absolute Value Equations Algebraically

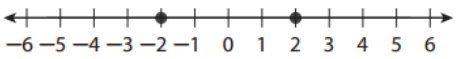
To solve absolute value equations algebraically, first isolate the absolute value expression on one side of the equation the same way you would isolate a variable. Then use the rule:

If $|x| = a$ (where a is a positive number), then $x = a$ OR $x = -a$

Notice the use of a **disjunction** here in the rule for values of x. You cannot know from the original equation whether the expression inside the absolute value bars is positive or negative, so you must work through both possibilities to finish isolating x.

Example 1 **Solve each absolute value equation algebraically. Graph the solutions on a number line.**

(A) $|3x| + 2 = 8$



Subtract 2 from both sides. $|3x| = 6$

Rewrite as two equations. $3x = 6$ or $3x = -6$

Solve for x. $x = 2$ or $x = -2$

(B) $3|4x - 5| - 2 = 19$

Add 2 to both sides. $3|4x - 5| = 21$

Divide both sides by 3. $|4x - 5| = 7$

Rewrite as two equations. $4x - 5 = 7$ or $4x - 5 = -7$

Add 5 to all four sides. $4x = 12$ or $4x = -2$

Solve for x. $x = 3$ or $x = -\frac{1}{2}$

Your Turn

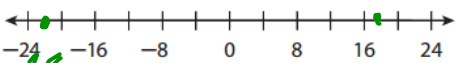
Solve each absolute value equation algebraically. Graph the solutions on a number line.

3. $\frac{1}{2}|x + 2| = 10$

$|x+2| = 20$

$x+2 = 20$ $x+2 = -20$

$x = 18$ $x = -22$



4. $-2|3x - 6| + 5 = 1$

$|3x-6| = 2$

$3x-6 = 2$

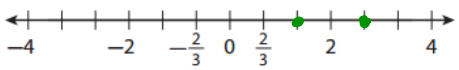
$3x = 8$

$x = 8/3$

$3x-6 = -2$

$3x = 4$

$x = 4/3$



Explain 2 Absolute Value Equations with Fewer than Two Solutions

You have seen that absolute value equations have two solutions when the isolated absolute value expression is equal to a positive number. When the absolute value is equal to zero, there is a single solution because zero is its own opposite. When the absolute value expression is equal to a negative number, there is no solution because absolute value is never negative.

Example 2 **Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."**

(A) $-5|x+1|+2=12$

Subtract 2 from both sides.	$-5\|x+1\|=10$
Divide both sides by -5.	$\|x+1\|=-2$
Absolute values are never negative.	No Solution

(B) $\frac{3}{5}|2x-4|-3=-3$

Add 3 to both sides.	$\frac{3}{5}\|2x-4\|=0$
Multiply both sides by $\frac{5}{3}$.	$\|2x-4\|=0$
Rewrite as one equation.	$2x-4=0$
Add 4 to both sides.	$2x=4$
Divide both sides by 2.	$x=2$

Your Turn

Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."

5. $3\left|\frac{1}{2}x+5\right|+7=5$

$-7 \quad -7$

$\frac{}{3} \quad \frac{}{3}$

$\left|\frac{1}{2}x+5\right|=-\frac{2}{3}$

No Solution

6. $9\left|\frac{4}{3}x-2\right|+7=7$

$-7 \quad -7$

$\frac{}{9} \quad \frac{}{9}$

$\left|\frac{4}{3}x-2\right|=0$

$\frac{4}{3}x-2=0$

$\frac{4}{3}x=2$

$\frac{3}{4}\left(\frac{4}{3}x=2\right)\frac{3}{4}$

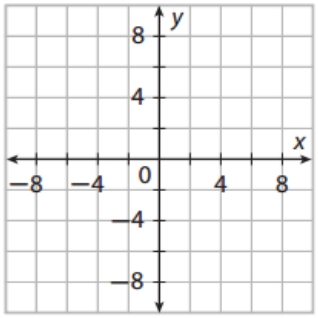
$x=\frac{3}{2}$

Pg 76 LPT, Pg 81 2-16 EVEN, 17, 18, 20

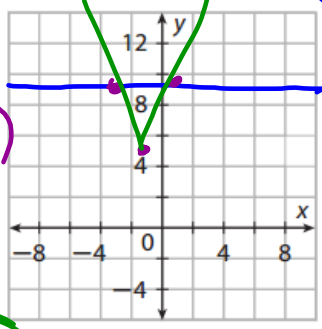
Evaluate: Homework and Practice

Solve the following absolute value equations by graphing.

1. $|x-3|+2=5$



2. $2|x+1|+5=9$

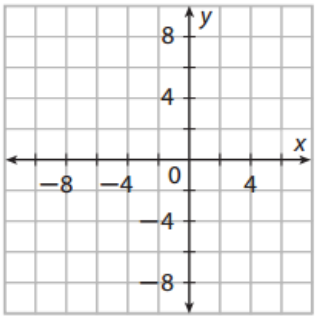


(2,2) → (1,9)

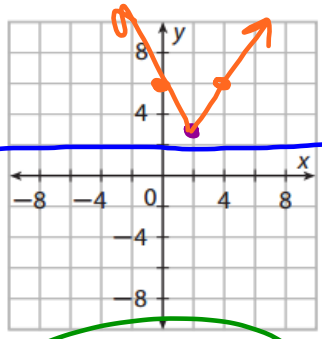
(-2,2) → (-3,9)

X=1, -3

3. $-2|x+5|+4=2$



4. $\left|\frac{3}{2}(x-2)\right|+3=2$



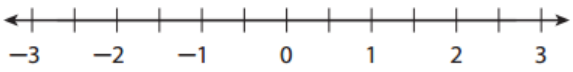
(3,3) → (4,6)

(-3,3) → (0,6)

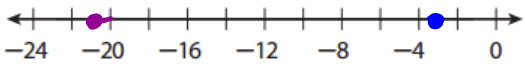
No Solution

Solve each absolute value equation algebraically. Graph the solutions on a number line.

5. $|2x|=3$



6. $\left|\frac{1}{3}x+4\right|=3$

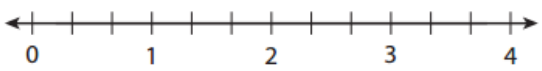


$\frac{1}{3}x+4=3$ $\frac{1}{3}x+4=-3$

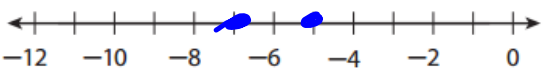
$\frac{1}{3}x=-1$ $\frac{1}{3}x=-7$

$x=-3$ $x=-21$

7. $3|2x-3|+2=3$



8. $-8|-x-6|+10=2$



$|-x-6|=1$

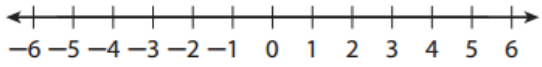
$-x-6=1$ $\quad -x-6=-1$

$-x=7$ $\quad -x=5$

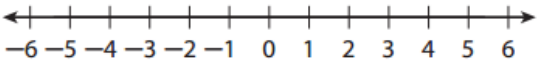
$x=-7$ $\quad x=-5$

Isolate the absolute value expressions in the following equations to determine if they can be solved. If so, find and graph the solution(s). If not, write "no solution".

9. $\frac{1}{4}|x+2|+7=5$

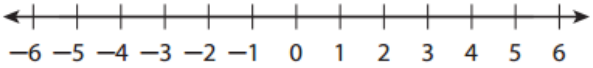


10. $-3|x-3|+3=6$

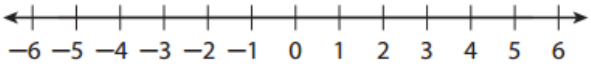


NO SOLUTION

11. $2(|x+4|+3)=6$



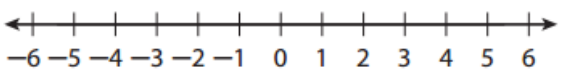
12. $5|2x+4|-3=-3$



$x=-2$

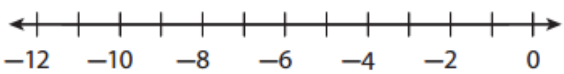
Solve the absolute value equations.

13. $|3x-4|+2=1$

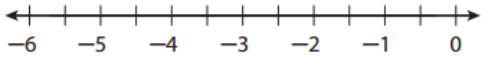


14. $7\left|\frac{1}{2}x+3\frac{1}{2}\right|-2=5$

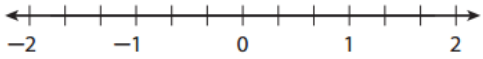
$-5, 9$



15. $|2(x+5)-3|+2=6$



16. $-5|-3x+2|-2=-2$



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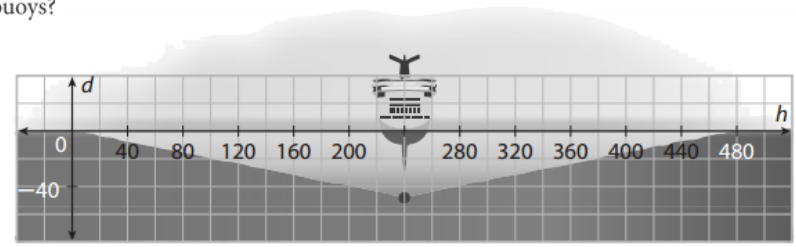
17. The bottom of a river makes a V-shape that can be modeled with the absolute value function, $d(h)=\frac{1}{5}|h-240|-48$, where d is the depth of the river bottom (in feet) and h is the horizontal distance to the left-hand shore (in feet).

A ship risks running aground if the bottom of its keel (its lowest point under the water) reaches down to the river bottom. Suppose you are the harbormaster and you want to place buoys where the river bottom is 30 feet below the surface. How far from the left-hand shore should you place the buoys?



$-30=\frac{1}{5}|h-240|-48$

$90=|h-240|$

$90=h-240$ $\quad -90=h-240$

$330=h$ $\quad 150=h$

150FT, 330FT

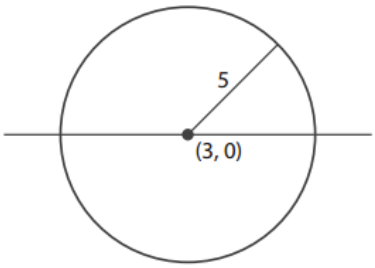
18. A flock of geese is approaching a photographer, flying in formation. The photographer starts taking photographs when the lead goose is 300 feet horizontally from her, and continues taking photographs until it is 100 feet past. The flock is flying at a steady 30 feet per second. Write and solve an equation to find the times after the photographing begins that the lead goose is at a horizontal distance of 75 feet from the photographer.

$|30t-300|=75$

$30t-300=75$ $\quad 30t-300=-75$

$t=12.5$sec $\quad t=7.5$sec

19. **Geometry** Find the points where a circle centered at (3, 0) with a radius of 5 crosses the x-axis. Use an absolute value equation and the fact that all points on a circle are the same distance (the radius) from the center.



20. Select the value or values of x that satisfy the equation $-\frac{1}{2}|3x - 3| + 2 = 1$.

A. $x = \frac{5}{3}$ B. $x = -\frac{5}{3}$

C. $x = \frac{1}{3}$ D. $x = -\frac{1}{3}$

E. $x = 3$ F. $x = -3$

G. $x = 1$ H. $x = -1$

$|3x-3| = 2$

$3x-3=2$ $3x-3=-2$

$3x=5$ $3x=1$

$x=\frac{5}{3}$ $x=\frac{1}{3}$

21. Terry is trying to place a satellite dish on the roof of his house at the recommended height of 30 feet. His house is 32 feet wide, and the height of the roof can be described by the function $h(x) = -\frac{3}{2}|x - 16| + 24$, where x is the distance along the width of the house. Where should Terry place the dish?

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** While attempting to solve the equation $-3|x - 4| - 4 = 3$, a student came up with the following results. Explain the error and find the correct solution:

$-3|x - 4| - 4 = 3$

$-3|x - 4| = 7$

$|x - 4| = -\frac{7}{3}$

$x - 4 = -\frac{7}{3}$ or $x - 4 = \frac{7}{3}$

$x = \frac{5}{3}$ or $x = \frac{19}{3}$