24.3 Graphing Square Root Functions

Essential Question: How can you use transformations of the parent square root function to graph functions of the form $f(x) = a\sqrt{x - h} + k$?

Explore 1 Exploring the Inverse of $y = x^2$

Use the steps that follow to explore the inverse of $y = x^2$.

(A) Use a graphing calculator to graph $y = x^2$ and $y = x$. Describe the graph.

(B) Use the DrawInv feature to graph the inverse of $y = x^2$ along with $y = x^2$ and $y = x$. Describe the new graph.

(C) State whether the inverse is a function. Explain your reasoning.

(D) Use inverse operations to write the inverse of $y = x^2$.

Switch x and y in the equation. $x = y^2$

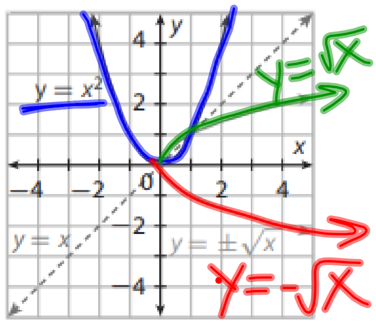
Take the square root of both sides of the equation. $\pm\sqrt{x} = y$

Reflect

1. **Discussion** Explain why the inverse of $y = x^2$ is not a function.

Explore 2 Graphing the Parent Square Root Function

The graph shows $y = \pm\sqrt{x}$, $y = x^2$, and $y = x$. You have discovered that $y = \pm\sqrt{x}$ is not a function. You will find out how to alter $y = \pm\sqrt{x}$ so that it becomes a function.



(A) For $y = \pm\sqrt{x}$ can x be negative? Explain your reasoning.

(B) If the domain of $y = x^2$ was restricted to $x \le 0$, would the inverse be a function? Explain your reasoning.

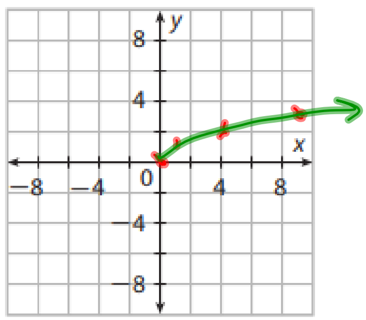
(C) If the domain of $y = x^2$ was restricted to $x \ge 0$, would the inverse be a function? Explain your reasoning.

(D) Typically the domain of $y = x^2$ is restricted to $x \ge 0$ before findng its inverse to create the parent square root function. What is the equation of the parent square root function?

(E) A **radical function** is a function whose rule is a radical expression. A **square root function** is a radical function involving $\sqrt{x}$.

Graph the parent square root function $y = \sqrt{x}$ by first making a table of values.

x	$y = \sqrt{x}$	(x, y)
0	$\sqrt{0}$	(0, 0)
1	$\sqrt{1}$	(1, 1)
4	$\sqrt{4}$	(4, 2)
9	$\sqrt{9}$	(9, 3)
16	$\sqrt{16}$	(16, 4)
25	$\sqrt{25}$	(25, 5)



(F) Plot the points on the graph and draw a smooth curve through them.

Explain 1 Graphing Translations of the Parent Square Root Function

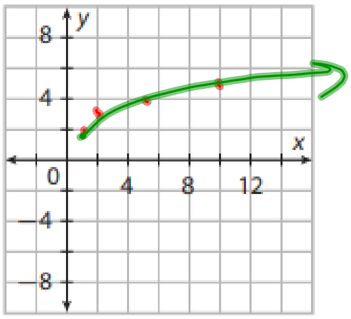
You discovered in Explore 2 that the parent square root function is $y = \sqrt{x}$. The equation $y = \sqrt{x - h} + k$ is the parent square root function with horizontal and vertical translations, where h and k are constants. The constant h will cause a horizontal shift and k will cause a vertical shift.

Example 1 **Graph each function by using a table and plotting the points. State the direction of the shift from the parent square root function, and by how many units. Then state the domain and range. Confirm your graph by graphing with a graphing calculator.**

(A) $y = \sqrt{x - 1} + 2$ → +1

0+1
1+1
4+1
9+1

x	$y = \sqrt{x-1} + 2$	(x, y)
1	$\sqrt{1-1} + 2$	(1, 2)
2	$\sqrt{2-1} + 2$	(2, 3)
5	$\sqrt{5-1} + 2$	(5, 4)
10	$\sqrt{10-1} + 2$	(10, 5)



The graph is translated 2 units up and 1 unit right.

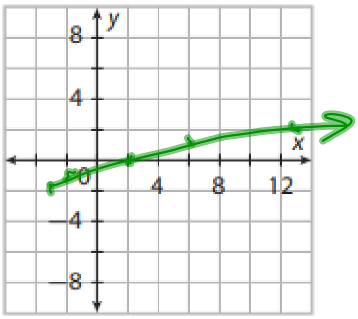
Domain: $x \geq 1$ Range: $y \geq 2$

(B) $y = \sqrt{x + 3} - 2$ → 2 DOWN

-3

0-3
1-3
4-3
9-3
16-3

x	$y = \sqrt{x+3} - 2$	(x, y)
−3	$\sqrt{-3+3} - 2$	(−3, −2)
−2	$\sqrt{-2+3}$ -2	(-2,-1)
1	$\sqrt{1+3}$ -2	(1, 0)
6	$\sqrt{6+3}$ -2	(6, 1)
13	$\sqrt{13+3}$ -2	(13, 2)



The graph is translated 2 unit(s) (up/down) and 3 unit(s) to the (left/right).

Domain: $-3 \leq x < \infty$ Range: $-2 \leq y < \infty$

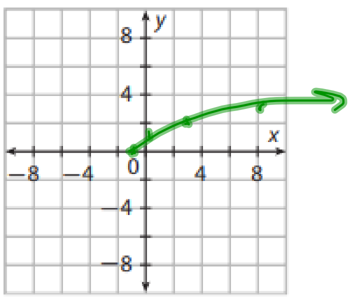
Your Turn

3. $y = \sqrt{x + 1}$

-1

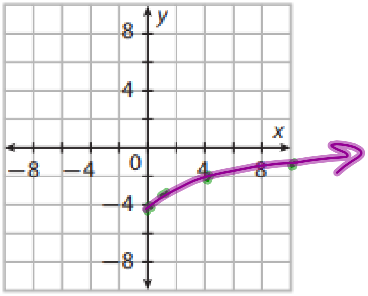
0-1
1-1
4-1
9-1

x	$y = \sqrt{x+1}$	(x, y)
−1		(-1, 0)
0		(0, 1)
3		(3, 2)
8		(8, 3)



4. $y = \sqrt{x} - 4$

x	$y = \sqrt{x} - 4$	(x, y)
0		(0, −4)
1		(1, −3)
4		(4, −2)
9		(9, −1)



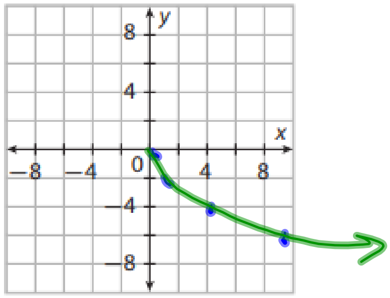
Explain 2 Graphing Stretches/Compressions and Reflections of the Parent Square Root Function

The equation $y = a\sqrt{x}$, where a is a constant, is the parent square root function with a vertical stretch or compression. If the absolute value of a is less than 1 the graph will be compressed by a factor of $|a|$, and if the absolute value of a is greater than 1 the graph will be stretched by a factor of $|a|$. If a is negative, the graph will be reflected across the x-axis.

Example 2 **Graph the functions by using a table and plotting the points. State the stretch/compression factor and whether the graph of the parent function was reflected or not. Then state the domain and range. Confirm your graph by graphing with a graphing calculator.**

(A) $y = -2\sqrt{x}$

x	$y = -2\sqrt{x}$	(x, y)
0	$-2\sqrt{0}$	(0, 0)
1	$-2\sqrt{1}$	(1, −2)
4	$-2\sqrt{4}$	(4, −4)
9	$-2\sqrt{9}$	(9, −6)

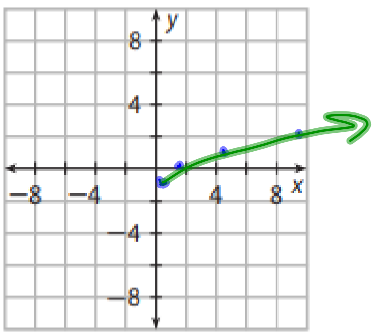


There is a vertical stretch by a factor of 2, and the graph is reflected across the x-axis.

Domain: $x \geq 0$ Range: $y \leq 0$

(B) $y = \frac{1}{2}\sqrt{x}$

x	$y = \frac{1}{2}\sqrt{x}$	(x, y)
0	$\frac{1}{2}\sqrt{0}$	(0, 0)
1	$\frac{1}{2}\sqrt{1}$	$(1, \frac{1}{2})$
4	$\frac{1}{2}\sqrt{4}$	(4, 1)
9	$\frac{1}{2}\sqrt{9}$	$(9, \frac{3}{2})$



There is a vertical (stretch/compression) by a factor of $\frac{1}{2}$,

and the graph (is/is not) reflected across the x-axis.

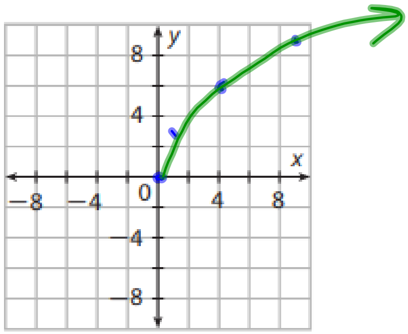
Domain: $0 \leq x < \infty$ Range: $0 \leq y < \infty$

Your Turn

Graph the functions by using a table and plotting the points. State the stretch/compression factor and whether the graph of the parent function was reflected or not. Then state the domain and range. Confirm your graph by graphing with a graphing calculator.

5. $y = 3\sqrt{x}$

x	$y = 3\sqrt{x}$	(x, y)
0	$3\sqrt{0}$	(0, 0)
1		(1, 3)
4		(4, 6)
9		(9, 9)



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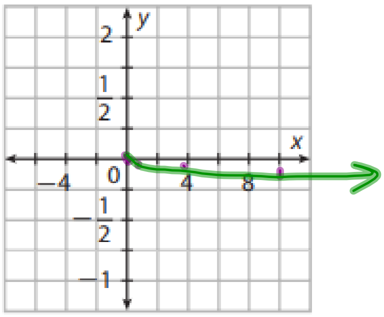
6. $y = -\frac{1}{4}\sqrt{x}$

x	$y = -\frac{1}{4}\sqrt{x}$	(x, y)
0	$-\frac{1}{4}\sqrt{0}$	(0, 0)
1		$(1, -\frac{1}{4})$
4		$(4, -\frac{1}{2})$
9		$(9, -\frac{3}{4})$



Explain 3 Modeling Real-World Situations with Square Root Functions

You can use transformations of square root functions to model real-world situations.

Example 3 **Construct a square root function to solve each problem.**

Ⓐ On Earth, the function $f(x) = \frac{6}{5}\sqrt{x}$ approximates the distance in miles to the horizon observed by a person whose eye level is x feet above the ground. Use the function to estimate the distance to the horizon for someone whose eyes are 6.5 ft above Earth's surface, rounding to one decimal place.

$f(x) = \frac{6}{5}\sqrt{x}$

$f(6.5) = \frac{6}{5}\sqrt{6.5}$ Substitute 6.5 for x.

$f(6.5) \approx 3.1$ miles Simplify.